

# MATHEMATICS

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Paper 9709/11  
Paper 11

## Key messages

Candidates should note the units for the range of functions in trigonometry questions and ensure their answers are expressed in these units.

When identifying the turning point it should be noted that the functions given will always have a second derivative which can be found using techniques within the syllabus.

Candidates should appreciate how the relationship between the range and domain of a function and its inverse can be used.

## General comments

All centres were provided with a 12 page booklet and additional sheets. Candidates should be aware there is no need to squeeze solutions into a few lines or omit what may be essential working. This point should be noted as the next series of exams will have the questions printed in the answer booklet.

## Comments on specific questions

### Question 1

**Part (i)** was well answered by many candidates and, for them, provided a straightforward start to the paper. It was expected that the result from **part (i)** would be used in **part (ii)** but this was rarely the case. Most preferred to restart using the quadratic equation:  $x^2 + 6x - 7 = 0$ . The answers  $-7$  and  $1$  were often left as the final answer or attached to an incorrect inequality.

Answers: (i)  $(x + 3)^2 - 7$  (ii)  $x < -7, x > 1$

### Question 2

Fewer candidates relied on producing the whole expansion and selecting the required term, favouring pre-selection of the appropriate term. Had brackets been used more effectively a lot of calculation errors would have been avoided.

Answer: 448

### Question 3

There were many good answers to **part (i)** using the correct expression,  $2r + 3r\alpha$ , for the perimeter. Some thought the shaded area was a sector. Those who attempted to work in degrees rarely reached the correct solution.

Of those able to set the difference in the two sector areas to 30 the squaring of  $2\alpha$  prevented some candidates reaching the correct final answer.

Answers: (i) 0.8 (ii) 5

#### Question 4

This area of coordinate geometry appeared to be well understood and many fully correct solutions were seen.

In **part (i)** it was rare to see alternative forms of the straight line equation used rather than the stated form. Most went on to use the distance formula correctly in **part (ii)** although there was some misuse of  $y = 0$  to find the coordinates of  $D$ .

Answers: (i)  $y = 2x - 10$       (ii)  $\sqrt{205}$

#### Question 5

Those who appreciated the need to express two sums in term of  $\alpha$  and  $r$  were more successful if the sums were written using separate terms, enabling them to eliminate  $a$  directly. Using the formula for the sum of  $n$  terms was often seen but very rarely successful.

Some potentially good solutions were spoiled by poor algebraic technique. A few candidates spotted that  $r = 30/50$  and were able to quickly reach the final answer.

Answer:  $625/8$

#### Question 6

The identity in **part (i)** was shown very quickly by using  $\cos^4 \theta = (1 - \sin^2 \theta)^2$ . The candidates who chose to write the right hand side in terms of  $\cos \theta$  took a little longer.

Using the result from **part (i)** in **part (ii)** was the easiest route to a simpler trigonometric equation in terms of  $\sin \theta$  although some chose to find a quadratic equation in  $\cos \theta$ . Both methods led to negative alternatives which were rarely considered.

Answers: (ii)  $35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ$

#### Question 7

The correct value of  $x$  in **part (i)** was reached very quickly through reference to the graph and substituting  $y = 0$  into either equation. Those who chose to solve a pair of simultaneous equations rarely completed the process.

Where it was attempted, the integration of  $(2x - 1)^2$  was done well in **part (ii)**. Some candidates chose to use  $\int x \, dy$  for the area under  $y^2 = 1 - 2x$  and used the correct limits successfully. The integration of  $(1 - 2x)^{1/2}$  proved to be quite challenging for many candidates especially the division by  $3/2$ .

Answers: (i)  $(1/2, 0)$       (ii)  $1/6$

#### Question 8

In **part (i)** the composite function was often set up correctly but dealing with  $4/g(x)$  proved a barrier to simplification. Few stated the range correctly and notation for range and domain was freely interchanged in both parts.

Finding the expression for the inverse of  $g$  in **part (ii)** was often carried out successfully and without the need for simplification, many fully correct answers were seen. Correct expressions for the domain were rarely seen. Only a few candidates realised that  $x = 0$  could not be part of the domain.

Answers: (i)  $fg(x) = 5x, fg(x) \geq 0$       (ii)  $g^{-1}(x) = (4 - 2x)/5x, 0 < x \leq 2$

### Question 9

In **part (i)** those who saw the need to find the scalar product of  $\overline{OP}$  and  $\overline{XP}$  were usually able to find both vectors and set the scalar product to zero to obtain the required quadratic in  $p$ . Attempts to use Pythagoras' theorem very rarely succeeded.

**Part (ii)** was the most well answered part of the question. Occasionally the magnitude was calculated and left as the final answer.

The candidates who found  $\overline{AG}$  correctly were often able to spot the ratio of  $\overline{AG} : \overline{XQ}$  from  $\overline{AB} : \overline{XB}$  and go on to find  $\overline{XQ}$ .

Answers: (i)  $p = 1, p = 4$  (ii)  $(1/6)(-4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$  (iii)  $(2/3)(-4\mathbf{i} + 15\mathbf{j} + 2\mathbf{k})$

### Question 10

The expected quadratic in  $\sqrt{x}$  was rarely seen in **part (i)** with most successful solutions coming from squaring  $3x - 2 = -\sqrt{x}$  and solving the resulting quadratic. The additional solution given by this method ( $x = 1$ ) was rarely discarded.

The integration of  $f'(x)$  was often seen in **part (i)** but only gained credit when used in **part (ii)**. Those who included a constant of integration were able to find an equation of the curve but only the few who had completed **part (i)** successfully were able to complete this **part (ii)**.

Answers: (i)  $4/9$  (ii)  $-2/27$

### Question 11

The candidates who attempted **part (i)** showed they appreciated the need to find  $\frac{dy}{dx}$  and most of them went on to find the equation of the normal. It was not always the case that  $y$  was set to zero to find the required coordinate. Those who were successful integrating the functions in **question 7** were adept in their differentiation of  $y$ .

As **part (ii)** was mainly dependent on a correct differentiation in **part (i)** fewer than expected completely correct answers were seen however the process stages were often carried out correctly. The use of the second derivative rather than a table of values of the first derivative gave the most correctly identified turning points. It should be noted that the use of the first derivative at  $x = 1$  was not considered part of a successful method to identify the turning point at  $x = 2$ .

Answers: (i) 13 (ii)  $-1$ , maximum; 2, minimum

# MATHEMATICS

Paper 9709/12  
Paper 12

## Key messages

Candidates would benefit from reading the questions carefully at least twice and then extracting the relevant information from them. Not fully comprehending the information given or making false assumptions often leads to a significant amount of time and effort being wasted and no worthwhile progress being made.

## General comments

The paper seemed to be generally well received by the candidates and many good and excellent scripts were seen. The paper was well balanced with a number of questions being reasonably straightforward, particularly near the beginning of the paper, giving all candidates the opportunity to show what they had learned and understood and some which provided more of a challenge, even for candidates of good ability. Some weaker candidates did struggle, however, from **Question 4** onwards. The vast majority of candidates appeared to have sufficient time to complete the paper and the standard of presentation was generally good with candidates setting their work out in a clear readable fashion.

## Comments on specific questions

### Question 1

The question proved to be an accessible start to the paper with a great many candidates realising the need to integrate in order to find the equation of the curve. Some struggled with the power of  $-\frac{1}{2}$  but the majority of candidates were able to obtain full marks. Very few candidates forgot to include or attempt to find  $+c$  but a small number did not integrate and instead used the equation of a straight line and thus received no credit.

Answer:  $y = 4\sqrt{4x+1} - 7$

### Question 2

Obtaining full marks for this question proved to be more difficult for candidates than expected. In part (i) some seemed confused by the constant  $k$  in the question but the vast majority knew that they needed to expand the brackets, collect like terms together and divide  $\sin 2x$  by  $\cos 2x$  in order to obtain  $\tan 2x$ . A great many candidates were then confident in part (ii) 'in finding the inverse tan before dividing by 2 and obtaining the solution in the first quadrant but very few were able to correctly identify the solution in the fourth quadrant.

Answers: (i)  $k = 3$  (ii)  $35.8^\circ, -54.2^\circ$

### Question 3

Very many candidates obtained full marks in both parts of this question. In **part (i)** a few candidates ignored the 13 given in the question and instead attempted to solve an equation which had no real solutions. Those who did take notice of the 13 were generally able to solve the resulting inequality correctly but some errors in factorising and identifying the correct regions did occur. In **part (ii)** the most common approach was to equate the curve and the line and then to use the discriminant. This was usually done successfully but some

weaker candidates struggled to correctly evaluate it with  $64 - 8(5 - k)$  sometimes becoming  $64 - 40 - 8k$ . An alternative approach was to differentiate the curve and equate it to 2, being the gradient of the line. Those who used this method were usually successful although some did equate the derivative to 0 and received no credit.

Answers: (i)  $x < -1, x > 4$       (ii)  $k = -3$

#### Question 4

This question tested candidates understanding of the binomial distribution in a slightly different way. This caused difficulty for some candidates who could quote general formulae but not apply them correctly in this case. Those who made progress in attempting to find the unknown quantity  $n$  were often hampered by failing to realise that the given coefficient of  $x$  and the coefficient of  $x^2$  would both come from two terms in this situation. Many fully correct solutions were seen though, and perhaps some candidates would benefit from more practice in this type of problem solving situation.

Answer:  $6, \frac{21}{4}$

#### Question 5

This question was found to be difficult by many candidates although a good number of fully correct solutions were seen. The main problem seemed to be that three distinct pieces of information were included in the question and often one or more of these were not used. This was particularly true for the middle one: that the mid-point of AB lies on the line  $2x + y = 10$ . A significant number of candidates ignored this information and attempted to solve the 2 given line equations simultaneously.

Answer:  $a = 6, b = 8$

#### Question 6

**Part (i)** was generally well answered although candidates need to be aware that given answers need to be fully justified. Some who used the cosine rule prematurely approximated the length of BD and then divided by two. These candidates were penalised along with those who failed to show all of the necessary steps in their working out. **Part (ii)** was generally well done although premature approximation again often meant that the final answer was not correct to three significant figures. Candidates would benefit from working to at least 4 significant figures to ensure that the final required accuracy is obtained. **Part (iii)** was found to be more challenging with many candidates not considering the whole of the required area. Those who realised that the areas of the sector, triangle and semicircle all needed to be evaluated and added were usually successful. Another approach was to subtract the area of the segment from the large circle and then add on the area of the small semicircle. There was some incorrect use of  $\pi - 1.2$  rather than  $2\pi - 1.2$  in **parts (ii)** and **(iii)**

Answers: (ii) 68.6      (iii) 351

#### Question 7

In **part (i)** the vast majority of candidates were able to differentiate correctly but many then failed to sufficiently explain why the curve had no stationary points in **part (ii)**. Those who set the derivative equal to 0 and showed that it could not be solved were generally most successful. **Part (iii)** was generally well done but some candidates failed to use the point  $P$  and simply used the origin in the equation of the line. Most candidates realised in **part (iv)** that the chain rule should be used but the majority failed to take account of the fact that the  $x$ -coordinate was decreasing.

Answers: (i)  $\frac{-6}{(2x-1)^2}$       (iv) 0.04

### Question 8

In **part (a)** many candidates failed to fully appreciate the situation described. The vast majority realised that an arithmetic series would occur but a significant number incorrectly used  $d = +5$  rather than  $d = -5$  because the distance covered each day was reducing. In **(a)(i)** many found the sum of the distances covered over the first 15 days rather than the distance covered on the 15th day. Candidates need to be aware that putting both answers down simply means that only the final answer is then marked. In **(a)(ii)** many correct quadratic equations were seen and solved although some candidates then incorrectly selected 61 rather than 20 as their answer. In **part (b)** two distinct pieces of information were given about the geometric progression but these were sometimes muddled up. Putting the 8 on the wrong side of the equation was a common error leading to the common ratio being 2. Candidates who obtained this answer often then worked out the sum to infinity without being concerned that  $r$  was greater than 1. Candidates should have realised that this indicated that a mistake has been made earlier in their working.

Answers: **(a)(i)** 130    **(ii)** 20 May    **(b)(i)** 16    **(ii)** 32

### Question 9

In **part (i)** the standard technique for finding the angle between two vectors was well understood by all but the weakest candidates. The correct vectors were almost always used, although occasionally one of them was reversed and more frequently candidates thought that the angle had to be acute and so subtracted the correct answer from  $180^\circ$ . Although in **part (ii)** the vast majority of candidates were able to find the vector **AC**, many of them seemed confused by the fact that the magnitude was meant to be 15. Some stopped after finding **AC** whilst a few multiplied **AC** by 15. In **part (iii)** some candidates failed to combine the two vectors into one and some of those who did then put the resultant equal to, rather than perpendicular to, **OB**. A minority thought that perpendicular meant that their scalar product would equal  $-1$ . A surprisingly large number of candidates omitted the negative sign when copying the  $z$  term of **OA** from the question. Many fully correct solutions were seen for the whole of this question but some candidates would benefit from doing more practice with questions which were different to the standard ones.

Answers: **(i)**  $139.6^\circ$     **(ii)**  $12\mathbf{j} + 9\mathbf{k}$     **(iii)**  $\frac{11}{4}$

### Question 10

A number of fully correct solutions were seen from better candidates but many others did not seem to appreciate the links between the different parts of this functions question. The answers to **parts (i) and (ii)** were often inconsistent with each other and only the best candidates appreciated what was required in **part (iv)** and its link with the inverse function. In **part (i)** many candidates substituted values for  $x$  in order to attempt to find the range. Often an insufficient number of  $x$  values were used so that values of 5 kept being calculated. Better candidates appreciated the need for values in between or realised that  $f$  could only range between 3 and 7 due to  $\sin 2x$  ranging between  $-1$  and  $1$ . Those who worked from the domain and replaced  $x$  with  $f$  in stages were usually successful. In **part (ii)** some straight lines were seen and a number of candidates omitted it, although the majority realised that a sinusoidal graph was required and many correct sketches were seen. In **part (iii)** most candidates correctly found the value of  $\sin x$  but many were then unable to work correctly with the negative value obtained from the calculator. **Part (iv)** was the worst done by candidates on the paper with only the best ones realising that  $g$  needed to be a one–one function in order to have an inverse and therefore only the first part of the sketch of  $f$  was allowed. **Part (v)** was more familiar to candidates and many correct solutions were seen although some failed to make their final answer sufficiently clear.

Answers: **(i)**  $3 \leq f(x) \leq 7$     **(iii)**  $\frac{7\pi}{12}, \frac{11\pi}{12}$     **(iv)**  $\frac{1}{2} \sin^{-1} \frac{(5-x)}{2}$

# MATHEMATICS

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Paper 9709/13  
Pure Mathematics 1

## Key Messages

The key messages in the June 2016 Report remain extremely relevant and these are repeated below in points 1 – 4.

- 1 Last November it was reported that a significant minority of candidates attempted to compress their work so that it occupied only two or three sheets of paper, a practice which can lead to the omission of essential working and, potentially, the loss of marks. It is pleasing to report that this practice was far less widespread in this examination session. Indeed, the introduction of standard answer booklets with at least 12 pages available, together with supplementary booklets if required, induced quite a number of candidates to be extremely extravagant with space by ignoring the printed lines and often leaving several printed lines between each line of their working. It should be noted, however, that it is expected that from May 2017 these Answer booklets will be replaced by Question/Answer booklets and, for each question or part question there will be a set amount of working space. This amount of working space will be more than adequate providing candidates use the printed ruled lines as a guide for setting out their working.
- 2 Notwithstanding the comment in 1 above, there are still candidates who are not showing sufficient working. This is particularly true in the case of questions involving direct integration. Candidates who do not show the actual result of the integration process will not, in general, score any marks.
- 3 In questions which require an *exact* answer it is necessary for candidates to employ an exact method. An inexact method which perhaps fortuitously rounds to the required answer will therefore not gain full marks. (See **Q.7(ii)**.)
- 4 Since scripts are now scanned and then marked on-line, examiners no longer have access to the paper script when marking but only the scanned image on their computer screens. This can occasionally cause difficulties for examiners when the candidate places a minus sign directly on a printed ruled line. Candidates should be advised to try to place minus signs between the printed ruled lines.
- 5 When candidates are required to reach a specific *given* answer, as in **Q.5(i)** and **Q.7(i)**, examiners need to be convinced that candidates have reached the required result legitimately, and hence no steps in the working should be omitted.

## General Comments

The paper was generally well received by candidates and many good scripts were seen. There were, however, fewer candidates with script totals in the range 70 – 75 and this was partly due to the fact that there were a number of places (even early on in the paper, e.g. **Q.1**, **Q.4**, **Q.8(iii)**) which caused even very good candidates to drop marks. Most candidates seemed to have sufficient time to finish the paper. In trigonometric questions, candidates should be aware that sometimes answers are required in degrees and sometimes in radians and it would be advisable for candidates, before embarking on the solution of the question, to check the required range and set their calculators to degrees or radians as appropriate.



### Comments on Specific Questions

#### Question 1

The vast majority were able to eliminate  $y$ , rearrange into a 3-term quadratic equation and find the discriminant, earning the first 2 marks. Very many candidates failed to score the final mark, often giving the answer as  $k > \pm 2$  or assuming that the correct region for  $k$  was between the critical values, giving the answer as  $-2 < k < 2$ . Candidates who drew a diagram as an aid to solving the inequality were generally the most successful.

*Answer:*  $k > 2, k < -2$ .

#### Question 2

This was a most successful question for candidates and the vast majority scored full marks. A small number of candidates forgot to raise  $a$  to the power 3, but most included the minus sign when evaluating the coefficient of  $x^3$  from the first expansion.

*Answer:*  $a = 4$ .

#### Question 3

Most candidates were able to use a relevant identity to transform the given equation into an equation in  $\tan^2 x$ , or  $\sin^2 x$  or  $\cos^2 x$ . A surprising number then forgot to square root. Of the remaining candidates many made an elementary error in forgetting the  $\pm$  sign. Some candidates lost marks by giving answers correct to 3 significant figures when angles in degrees need to be given correct to 1 decimal place.

*Answer:*  $50.8^\circ, 129.2^\circ, 230.8^\circ, 309.2^\circ$ .

#### Question 4

The vast majority of candidates started their response correctly by finding the derivative and the majority went on to set this either equal to zero or to greater than zero, producing critical values for  $x$  of 3 and  $-1$ . At this point many candidates, realising they had to formulate their answers in terms of the least value of  $n$ , got confused. Some chose the wrong critical value ( $-1$ ) whilst others gave the answer as 4. Other candidates, having found the derivative, went on to complete the square or to find the second derivative and set this to zero – both approaches often finishing with 1 as the final answer.

*Answer:* The least possible value of  $n$  is 3.

#### Question 5

Part (i) of the question was intended as a straightforward lead into part (ii) but many candidates failed to see that all that was required was to find  $6\cos(0.9)$ . (Note that the omission of showing this key calculation after writing  $\cos(0.9) = OE/6$ , lost one of the 2 marks for this part of the question since the answer is given.) A significant number of candidates used very long methods (e.g. cosine rule followed by Pythagoras) to achieve the same result.

For part (ii) most candidates used the direct method of finding the area of the two sectors - although some candidates were clearly uncomfortable with using  $(2\pi - 1.8)$  and attempted more indirect ways of finding the area of the major sector.

*Answers (i) 3.73; (ii) 93.2.*



### Question 6

A majority of candidates obtained the correct coordinates for the point  $C$  in part (i). Those who were not successful often misunderstood what was required and found the mid-point of  $AB$ . A surprising variety of methods were seen for part (ii), many methods being quite long and complicated and often not resulting in a correct answer or any answer at all. The most direct method was to aim at producing two equations in  $m$  and  $n$ , the first equation coming from putting the gradient of  $AB$  equal to  $-1$  and the second from substituting the coordinates of  $C$  into the equation  $y = x + 1$ . This proved to be quite a challenging question.

Answers: (i)  $(2n - 2, -12 - m)$ ; (ii)  $m = -9, n = -1$ .

### Question 7

In part (i) it is necessary to show clearly only that each of the three scalar products is zero. Despite the fact that the question said, ".....showing all necessary working ....", some candidates unfortunately simply stated they were zero without showing the required arithmetic leading to zero in each case, whilst others neglected to state a conclusion. Part (ii) was a good source of marks for many candidates. However, some candidates assumed angle  $ACB$  was  $90^\circ$ , despite being told in the question that angle  $CAB$  was  $90^\circ$ . Other candidates did not recognise the significance of being asked to find the exact values of the area of the triangle and volume of the pyramid and thereby did not gain full marks.

Answers: (ii) Area of triangle  $ABC = \frac{1}{2}\sqrt{66}$ , Volume of pyramid = 11.

### Question 8

In part (i) many candidates lost marks by choosing to give their answer in the form  $a(x + b)^2 + c$  rather than  $(ax + b)^2 + c$ . Candidates who equated coefficients between the given expression and the required expression were often the most successful. In part (ii), even candidates who had the correct form from part (i) sometimes went back and found  $g(x)$  from the start again. Others correctly found  $g(x)$  from first principles not realising they already had the structure from part (i). Unnecessarily time-consuming work can often be avoided by checking the mark allocation. In this particular case an allocation of 1 mark for part (ii) is an indicator that recognition rather than detailed work is appropriate. In part (iii), most candidates successfully used their answer to part (i) to obtain the inverse function. Many candidates did not show the  $\pm$  sign for the square root – but on this occasion the  $+$  sign was the correct sign which meant that some candidates gained the mark somewhat fortunately by default. Finding the domain proved to be beyond most candidates.

Answers: (i)  $(2x + 3)^2 + 1$ ; (ii)  $2x + 3$ ; (iii)  $[\sqrt{(x - 1) - 3}]/2$ , for  $x > 10$ .

### Question 9

It was anticipated that part (a) would be a good source of marks for most candidates - but this did not altogether prove to be the case with a surprising number trying to solve  $6/(1 - r) = 12/(1 - r)$ . Of those that had the initial equation right and successfully found the value of  $r$ , a surprising number stopped at this point having not read the question thoroughly which required the sum to infinity. Part (b) was usually well attempted with significant numbers achieving full marks. Almost all candidates used the correct formula for the sum of an arithmetic progression, with the majority able to simplify to a 3-term quadratic equation in cosine and find the two solutions. However some candidates lost marks by giving  $\theta$  correct to 2 significant figures whilst others gave answers in degrees rather than radians.

Answers: (a) 9; (b) 0.841, 2.09.

### Question 10

The question discriminated successfully and there was a significant number of excellent answers that were completely correct. Weaker candidates, however, showed considerable confusion between the first two parts of the question; it appeared that, too often, candidates viewed the given  $dy/dx$  and decided that this required integrating immediately rather than carefully considering what the question was actually requiring them to do. Having integrated in part (i), a significant proportion of these candidates went on to solve part (i) of the question as part (ii) and were then substituting into their equation of the tangent by the time they reached part (iii). Although some of these attempts involved accurate algebra throughout, the overriding requirement is to answer the question set and these attempts failed to do this.

In part (i), those who correctly recognised that the gradient of the tangent was available by substituting  $x = a^2$  into the given expression for  $dy/dx$  were then faced with combining the two terms in  $a$ . When correct the two terms should be expressed in terms of either  $a^{-2}$  or  $1/a^2$ . Those aware that a line through the origin was of the form  $y = mx$  were able to quickly arrive at the required answer but most chose to write  $y - 3 = 3/a^2(x - a^2)$  arriving at the required result by a slightly slower route. This part was generally not done well and completely correct solutions were rare.

In part (ii), apart from the candidates who had muddled the first two parts, the vast majority appreciated that integration of  $dy/dx$  was required. Unfortunately, not all realised that a constant of integration was necessary and were unable to access the last 2 marks in this part and 3 of the 5 available marks in part (iii). It was pleasing to see that many were able to handle the algebra successfully in order to find the correct expression for  $y$ .

In part (iii), candidates who had obtained a correct expression for  $y$  in part (ii) were often able to produce a fully correct answer for this part. The question required the substitution of the coordinates (16,8) into their  $y$  expression and the solution of the resulting quadratic equation. It was of some concern to see a number of solutions where the required steps were not shown in detail. Aside from the importance of showing detailed working, it can prove very costly if small slips are made

Answers: (i)  $y = \frac{3}{a^2}x$ ; (ii)  $y = \frac{4x^{1/2}}{a} - 2ax^{-1/2} + 1$ ; (iii)  $a = 2$ ,  $AB = 13$

### Question 11

In part (i), most candidates followed the basic method for finding stationary values through the use of setting the first derivative to zero. However, whilst many candidates differentiated  $(kx - 3)^{-1}$  perfectly (some forgetting the factor  $k$ ), a surprising number of errors were made in differentiating the second term of the function,  $(kx - 3)$ , almost certainly induced by the presence of brackets round  $kx - 3$ . Those with the correct derivative were usually able to reach  $(kx - 3)^2 = 1$  but a disappointing number subsequently forgot the  $\pm$  and so obtained only one stationary value. The vast majority of candidates attempted to find the second derivative in order to determine the nature of the stationary values but earlier errors sometimes meant that the second derivative was not accurately found which, in these circumstances, usually prevented the award of the last 3 marks.

In part (ii), the use of  $\pi \int y^2 dx$  was generally well understood. When the function was left in the given form, squaring it was usually completed successfully (although frequently the term 2 was omitted). The subsequent integration then rarely caused a problem. However, if candidates attempted to rearrange the function before squaring (e.g. expressing the function over a single denominator  $x - 3$ ) little or no progress was usually made with integrating the resulting function. In order to gain credit candidates had to show clearly the integration process and the use of the limits 0 and 2. Those who produced largely unsupported answers should have appreciated what constitutes 'necessary working'.

Answers: (i)  $x = 2/k$  maximum,  $x = 4/k$  minimum; (ii)  $40\pi/3$ .

# MATHEMATICS

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Paper 9709/21  
Paper 21

## Key messages

Candidates should ensure that they are working to the required level of accuracy as specified on the front of the examination paper. Candidates should also think about the 'reality' of their answers, for example obtaining the logarithm of a negative number should raise awareness of an error in previous work. Parts in questions are usually related with later parts often requiring the use of work done in an earlier part of the question.

## General comments

The cohort taking this paper was small, so generalisations are difficult to make. The report will identify how questions were expected to be answered when specific problems or strong points cannot be identified. Time did not appear to be an issue although many candidates showed a lack of understanding of the syllabus objectives and would benefit from working through more past papers.

## Comments on specific questions

### Question 1

- (i) Very few candidates recognised that the equation was a quadratic equation in  $3^x$  and were thus unable to obtain a valid value for either  $3^x$  or subsequently  $x$ .
- (ii) It was intended that candidates recognise that the solution to this part was the positive and negative value of their answer to **part (i)**. This part was usually either omitted by candidates or the question was started again, even though the mark allocation was one mark. Candidates need to be aware of the implication of the word 'Hence' when used in this context.

Answer: (i) 1.77      (ii)  $\pm 1.77$

### Question 2

It was intended that candidates re-write the given equation in logarithmic form and either make use of the gradient of the line or use a method of substitution to obtain  $A$  and  $p$ . Many candidates showed a limited understanding of the principles involving logarithms, making incorrect use of  $\ln 4.77$  and  $\ln 3.17$  having reduced the given equation to logarithmic form. Those candidates who chose to make use of the original equation and express  $y$  as an exponential were often unable to eliminate  $A$  correctly in order to find  $p$ .

Answer:  $p = 0.32$      $A = 4.81$

### Question 3

Although candidates attempted to differentiate, completely correct differentiation was rare, with many candidates leaving the 6 in as part of their result. Most attempted to equate their result to zero, but few simplified to an equation in  $\tan 2x$ . Of those candidates that did, answers in degrees rather than radians were given, in spite of the range of  $x$  being given in terms of  $\pi$ .

Answer: 1.38, 2.95

#### Question 4

- (i) Many candidates did not realise that they were expected to integrate the given expression and just substituted in limits to  $4e^{2x} + 5$ . Those that did integrate and make a correct substitution of limits were often unable to manipulate their result to the required form.
- (ii) This part was often not attempted by candidates if they had not obtained the required result in **part (i)**. Centres should encourage candidates to use a given answer in a subsequent part even if they have been unable to obtain a given answer. Any errors in the iterations were usually due to an incorrect form entered on the calculator. Candidates should check the validity of their responses. If a sequence of iterations is diverging rather than converging then a check on the form being used should be made.

Answer: (ii) 1.854

#### Question 5

- (i) It was intended that candidates make use of the double angle formula and obtain the left hand side of the given expression in terms of  $\cos x$ , thus enabling factorisation and hence simplification to the given result. Too many candidates attempted this question by multiplying both sides of the identity by  $\cos x + 4$ .
- (ii) Candidates should be aware of the implications of the word 'Hence' when used in mathematical questions of this kind. It was intended that candidates make use of the result in **part (i)** to integrate. Of the candidates that did realise this,  $2\cos x + 1$  was used rather than  $2\cos 2x + 1$ .

Answer: (ii)  $2\pi$

#### Question 6

Most candidates were able to attempt implicit differentiation and gain some marks. Errors in the differentiation of the product were common as was failure to differentiate 24 with respect to  $x$ . Many made attempts at the equation of the normal, but all too often failed to give the answer in the required form.

Answer:  $5x - 9y + 22 = 0$

#### Question 7

- (i) Most candidates were able to obtain at least one correct equation using either the factor theorem or the remainder theorem and make use of simultaneous equations to find the value of  $a$  and of  $b$ .
- (ii) It was intended that candidates make use of the discriminant of the resulting quadratic equation after either factorisation or long division. Not many candidates did this and often ended up with more than one real root. Candidates should be encouraged to go back and check their work for errors if they do not obtain the expected result.
- (iii) Very few candidates attempted this part, having problems with relating  $\sec x$  to the linear factor given in **part (i)** and made use of in **part (ii)**.

Answer: (i)  $a = 2$   $b = 5$  (iii)  $\pm 109.5^\circ$

# MATHEMATICS

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Paper 9709/22  
Paper 22

## Key messages

Candidates should ensure that they are working to the required level of accuracy as specified on the front of the examination paper. Candidates should also think about the 'reality' of their answers, for example obtaining the logarithm of a negative number should raise awareness of an error in previous work. Parts in questions are usually related with later parts often requiring the use of work done in an earlier part of the question.

## General comments

Well prepared candidates were able to show their understanding of the syllabus aims and objectives by making very good attempts at all the questions. There appeared to be no timing issues. However, there were a number of poorly performing candidates, many of whom showed little evidence of preparation for the examination or little understanding of the syllabus objectives. Basic errors in algebraic manipulation were common as well as mathematical misinterpretation.

## Comments on specific questions

### Question 1

Most candidates chose to use two simple linear equations successfully to obtain the required solutions. Candidates who chose to attempt to square both sides of the equation tended to be less successful as they often failed to either square the right hand side of the equation or make algebraic errors in the expansion of the brackets, factorisation or use of the quadratic formula.

*Answer:* -3, 7

### Question 2

- (i) This question was rarely done correctly with candidates failing to recognise that  $4^y$  could be written as  $2^{2y}$  which enabled reduction to a quadratic equation. Candidates that did get a value of  $2^y$  correctly, often failed to discard the negative value.
- (ii) Many candidates failed to realise that the first part of the question was to be used to answer this part and often started the question again, usually with the same result as **part (i)**. Attempts to take the logarithm of a negative number were too common, hence the need for candidates to question the validity of their answers and check for previous incorrect work.

*Answer:* (i) 7 (ii) 2.81

### Question 3

- (i) Many correct solutions were seen, probably helped by the fact that the required answer was given, thus enabling candidates to think carefully about their work and identify errors. There were some contrived answers and some candidates, having integrated the exponential term correctly, forgot to integrate the constant term.
- (ii) The sketch of the required graph was not often done correctly with many candidates choosing to plot points. It was expected that candidates be aware of the nature of exponential curves, not have to resort to joining a series of points to obtain a far from smooth curve or in some cases a straight line. The labelling of the intercept on the y-axis was not required in the question, but if candidates chose to mark it in, it should be correct. Many graphs had an intercept at (0,3) rather than (0,7).
- (iii) Many correct conclusions were reached as to whether or not an estimate using the trapezium rule was an overestimation or an under-estimation, but very few were justified correctly. It was expected that candidates should mention that the trapezia used in the estimation were such that the area under the curve lay beneath the tops of the trapezia, or words to that effect.

Answer: (iii) More than

### Question 4

- (i) A question that most candidates were able to score some marks on, there were still candidates who in spite of being told to use the factor theorem, chose to use algebraic long division or form an identity. Many candidates did not appreciate that it did not matter what value  $a$  took and attempted to find a value for  $a$ . Some candidates looked at different values of  $a$  and made use of the factor theorem for their values. Candidates were expected to make a conclusion after the application of the factor theorem but this was often omitted.
- (ii) Most candidates were able to make the correct use of the remainder theorem and apart from a few arithmetic slips, were able to obtain the required solution.
- (iii) Rarely done well, it was intended that  $p(x)$  should be factorised first, followed by a substitution of  $x^2$  for  $x$  and further factorisation. Correct factorisation of  $p(x)$  was often seen but few candidates were able to deduce the final step. Attempts to factorise equations of the order 6 were common with candidates choosing to substitute  $x^2$  for  $x$  initially. These attempts were not successful.

Answer: (ii)  $-13$  (iii)  $(x^2 + 1)(2x - 1)(2x + 1)(x - 2)(x + 2)$

### Question 5

- (i) Most candidates recognised the need to differentiate using the quotient rule, usually successfully. Rearrangement to the required form was less successful and showed a lack of understanding of logarithms together with a weakness in the application of general algebraic techniques.
- (ii) This part was very often not attempted by candidates who had failed to obtain the required result in **part (i)**. Centres should encourage candidates to be aware of questions where a given answer can be used in later parts of a question in spite of not achieving the required result initially. For those candidates that did attempt the iterations, errors were often made with entering the form required on their calculators. Again, candidates should question the validity of their answers. If a sequence of iterations is diverging rather than converging then a check should be made on the form entered on the calculator.

Answer: (ii) 1.895

### Question 6

- (i) Many candidates were able to obtain a mark for the use of the same double angle formula in both the numerator and denominator of  $\frac{\cos 2\theta}{1 + \cos 2\theta}$ , but then failed to realise that very straightforward manipulation was needed to obtain the required result. Attempts at solutions involving both  $\sin \theta$  and  $\cos \theta$  were common, with differing levels of success.
- (ii) Provided candidates recognised that they needed to use  $\tan^2 \alpha + 1 = \sec^2 \alpha$  together with the result from **part (i)**, many were able to obtain marks recognising that a quadratic equation in  $\tan \alpha$ . Some candidates failed to realise that solutions needed to be in terms of radians even though the range for  $\alpha$  was given in terms of  $\pi$ . Those candidates who chose to attempt a solution using  $\sin \alpha$  and  $\cos \alpha$  were unsuccessful, being unable to obtain an equation in just one trigonometric ratio. Candidates should be encouraged to consider other methods of solution if they are unable to reach a workable equation after a few minutes.
- (iii) Correct solutions were rare, with many candidates failing to realise that they were meant to make use of **part (i)** to help with the integration. Of those candidates that did attempt to make use of **part (i)** many failed to recognise that they needed to integrate  $1 - \frac{1}{2}\sec^2 \frac{x}{2}$  rather than  $1 - \frac{1}{2}\sec^2 x$ .

Answer: (ii) 1.77 (iii)  $\pi - 2$

### Question 7

- (i) Of those candidates that attempted this question, most were able to score marks in this part. Most attempted to differentiate the given parametric equations with respect to  $\theta$  to obtain  $\frac{dy}{dx}$ , with varying levels of success. Many failed to use the compound angle formula correctly, if at all. Algebraic and arithmetic slips meant that few got the correct value for  $k$ .
- (ii) Many candidates did not attempt this part, especially if they had not obtained a value for  $k$ . Of those candidates that did, most were able to gain credit for correct methods used. Candidates should also be aware of the meaning of the word 'exact' in the context of the question as some candidates lost the final mark having given  $m$  and/or  $c$  in decimal form.

Answer: (i)  $k = -\frac{3}{8}$  (ii)  $y = \frac{8}{3}x + 1 + \frac{3\sqrt{3}}{2}$



# MATHEMATICS

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Paper 9709/23  
Paper 23

## Key messages

Candidates should ensure that they are working to the required level of accuracy as specified on the front of the examination paper. Candidates should also think about the 'reality' of their answers, for example obtaining the logarithm of a negative number should raise awareness of an error in previous work. Parts in questions are usually related with later parts often requiring the use of work done in an earlier part of the question.

## General comments

The cohort taking this paper was small, so generalisations are difficult to make. The report will identify how questions were expected to be answered when specific problems or strong points cannot be identified. Time did not appear to be an issue although many candidates showed a lack of understanding of the syllabus objectives and would benefit from working through more past papers.

## Comments on Specific Questions

### Question 1

- (i) Most candidates were able to make use of their calculator correctly, giving the iterations appropriately.
- (ii) Very few candidates realised the implication of the word exact, choosing to give their answer in decimal form.

Answer: (i) 2.289 (ii)  $\sqrt[3]{12}$  or  $12^{\frac{1}{3}}$

### Question 2

It was intended that candidates re-write the given equation in logarithmic form and either make use of the gradient of the line or use a method of substitution to obtain  $K$  and  $p$ . Many candidates showed a limited understanding of the principles involving logarithms, making incorrect use of  $\ln 1.28$ ,  $\ln 2.11$ ,  $\ln 3.69$  and  $\ln 4.81$  having reduced the given equation to logarithmic form.

Answer:  $p = 1.35$   $K = 7.11$  or  $7.12$

### Question 3

- (i) It was intended that candidates should make use of the identity  $\tan^2 4x + 1 \equiv \sec^2 4x$  in order to obtain an integrand for which a standard result could be used.
- (ii) A standard question testing integration of trigonometric ratios, candidates were expected to make use of their knowledge of the trigonometric ratios of standard angles and obtain an exact result.

Answer: (i)  $\frac{1}{4} \tan 4x - x + c$  (ii)  $3 - \sqrt{2}$

#### Question 4

- (i) Most candidates were able to use the factor theorem correctly and obtain a value for  $a$ .
- (ii) Many candidates reached the result  $(2x - 1)(x^2 + 5x + 2) = 0$  and stopped, saying that the quadratic equation had no real roots. It was intended that candidates preferably make use of the discriminant or attempt to solve  $x^2 + 5x + 2 = 0$  and reach an appropriate conclusion. The important part in the wording of this question is the word 'show'. Most did not show that the quadratic equation did not have any real roots.
- (iii) Most realised that a simple substitution of  $6^y$  for  $x$  was needed initially followed by an appropriate method to obtain  $y$ .

Answer: (i)  $a = 2$  (iii)  $-0.387$

#### Question 5

- (i) Some candidates had problems working out required ordinates using the given equation, but most were able to attempt a correct method of application of the trapezium rule.
- (ii) Not enough explanation was given by most candidates. It is simply not enough to say that the curve is either 'concave up' or 'concave down' or similar. Reference to the trapezia and where they lie with respect to the curve must be made.
- (iii) Reasonable attempts at integration of the  $1 + e^{\frac{1}{3}x}$  were made, however some candidates thought that the limits were 0 and  $2\pi$  rather than the required 0 and 6. Again candidates should be aware of the meaning of the word 'exact' in the context of these type of examination questions. Too many candidates make use of a calculator in these type of questions.

Answer: (i) 12.25 (iii)  $\pi(3 + 3e^2)$

#### Question 6

- (i) Most candidates recognised that they needed to differentiate each of the given equations with respect to  $t$ , but many failed to recognise that  $y$  was a product of two functions of  $t$  and thus needed to be differentiated as a product. The application of the chain rule was usually correct.
- (ii) It was intended that candidates equate their result from (i) to zero and attempt to solve. Some gave more than one value for  $t$ , usually including  $t = 0$ , not heeding the wording of the question which implies that there is only one turning point to find.
- (iii) A relatively easy part to the question in which candidates were expected to find the value of  $t$  when  $y = 0$  and use this in their expression for  $\frac{dy}{dx}$ .

Answer: (i)  $(t + 1)(2t \ln t + t)$  (ii)  $t = e^{-\frac{1}{2}}$  (iii) 2

**Question 7**

- (i) For those candidates that attempted this question, it was a straightforward expansion of brackets and simplification of the various trigonometric ratios.
- (ii) A standard piece of mathematics provided candidates had obtained a correct form in **part (i)** and made use of it. Some candidates seemed unaware of the implication of the word 'Hence' and attempted to start the question again.
- (iii) Of those candidates that attempted this part of the question, few obtained both solutions even though correct approaches were made. There were some problems with accuracy. Angles in degrees need to be given to one decimal place unless otherwise stated.

*Answer.* (i)  $6\sin \theta + 8\cos \theta$       (ii)  $R = 10$     $\alpha = 53.1$       (iii)  $82.4^\circ$     $351.3^\circ$

# MATHEMATICS

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Paper 9709/31  
Paper 31

## General comments

The candidates found aspects of this paper quite challenging. Whereas most candidates demonstrated knowledge and understanding of the majority of topics, all too frequently solutions went wrong due to algebraic and arithmetic errors. Many candidates scored well on **Question 2** (binomial expansion), **Question 3** (trigonometric equation), **Question 6(iv)** (application of an iterative formula) and **Question 8(i)** (perpendicular planes). The response to **Question 1** (algebra and indices) was particularly disappointing; candidates did not seem to pause and reflect before working through the simple algebraic processes required. Other questions that candidates found challenging were **Question 4** (implicit differentiation), **Question 5(ii)** (integration of  $\tan \theta$ ), and **Question 10(i)** (solution of a differential equation).

In **Question 5(i)**, **Question 5(ii)**, **Question 6(i)**, **Question 6(iii)**, **Question 8(i)** and **Question 10(i)** candidates were asked to demonstrate a given answer. It is important for candidates to realise that in these circumstances they need to show full reasoning to support their conclusions in order to score full marks. Similarly, when a question says '... the use of a calculator is not permitted' or asks candidates to 'find the exact value...' this is a clear message to candidates that an answer copied from a calculator will not be sufficient to score the marks available.

Candidates should read through the questions carefully and ensure that their solution answers the question. In this paper this was a particular issue in **Question 2**, **Question 6(i)**, and **Question 10(i)**.

Candidates preparing for this paper should be aware of how easy it is to lose marks through algebraic and arithmetic errors. In particular, many candidates demonstrate poor and inaccurate use of brackets. A disciplined approach to algebra and accurate use of the notation is an important part of mathematical communication. Similarly, candidates need to be aware of the difference between 'significant figures' and 'decimal places' and to be able to use these accurately.

The standard of presentation of the work was often very good, and the best candidates made their reasoning clear by showing full working. With the advent of the marking of scanned scripts, candidates should be aware that if they erase work and write over it then the result is barely legible for the examiner – it is far better to cross through the original and replace it. When candidates have used additional answer booklets it is helpful if these are attached in the correct order.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

### Comments on specific questions

#### Question 1

There were many concise and correct solutions to this question. The most common errors involved incorrect statements such as  $8 \times 3^x = 24^x$  and  $24^x - 3^x = 21^x$ . Some candidates are so accustomed to solving modular inequalities in **Question 1** that they set about squaring the given equation. Another popular tactic was to multiply top and bottom of the fraction by  $3^x + 2$ . Those with strong algebra were able to work through the additional work that this created. This question required an answer 'correct to 3 decimal places'; some candidates truncated their answer rather than rounding it, or gave only 2 decimal places in their answer.

0.860

#### Question 2

Most candidates demonstrated a good understanding of how to use the binomial expansion for a rational index. Many solutions started with a correct unsimplified expansion of  $(1 + 2x)^{\frac{3}{2}}$  as far as the term in  $x^2$ . Simplified expressions were often incorrect due to slips in the arithmetic in dealing with  $(2x)^2$ . Some candidates used powers of  $x$  rather than of  $2x$ . A few candidates also attempted to expand  $(2 - x)$ , however most of them did obtain  $(2 - x)$  as their answer. Many candidates lost marks in this question due to errors in their algebra and arithmetic when multiplying their expansion by  $(2 - x)$ .

There was a common misunderstanding that the question was only asking for the term in  $x^2$  rather than terms 'up to and including the term in  $x^2$ '.

$2 - 7x + 18x^2$

#### Question 3

Access to this question depended on knowing the definition of  $\sec \theta$  followed by correct use of  $\cos^2 \theta + \sin^2 \theta = 1$ . Most candidates were successful in reaching a correct quadratic in  $\sin \theta$ . Although the quadratic was straight forward to factorise, several candidates preferred to use the quadratic formula. Candidates should be aware that if they do not state the formula correctly and make an error in applying it then this will be regarded as a method error. Many candidates did reach the correct values of  $\sin \theta$ . However, after extensive working they often rejected or did not consider the only solution within the required range.

$-41.8^\circ$

#### Question 4

Many candidates found this question challenging. Some candidates tried to avoid implicit differentiation through attempting to rearrange the equation before differentiating, however this was not possible in this case.

The most common approach was to multiply out the brackets and differentiate the two terms separately. Some errors crept in at this stage with not all candidates reaching  $x^2y - 6xy^2$  and those who did, often had a sign error when combining the derivatives of the two terms. Many candidates differentiated the constant term  $9a^3$  to obtain  $27a^2$  making further progress impossible. Only a small number of candidates substituted  $\frac{dy}{dx} = 0$  before making  $\frac{dy}{dx}$  the subject of an equation. Many got to an expression for  $\frac{dy}{dx}$  and went no further. Some thought that for the tangent to be parallel to the axis then either  $x = 0$  or  $y = 0$ . Only a minority of the candidates who reached a correct expression in  $x$  and  $y$  explicitly rejected the possibility of  $y = 0$ .

$$(-3a, -a)$$

#### Question 5

- (i) Many candidates made good progress in tackling the identity, although several took very circuitous routes. The most common errors involved losing the denominator when combining the two terms on the left hand side, and sign errors in removing brackets (which were often omitted). Some candidates appeared to confuse  $\tan 2\theta - \tan \theta$  with  $\tan(2\theta - \theta)$ .
- (ii) Many candidates did not see the link between the two parts of this question. Of those who did, the majority did not recognise the standard integral of  $\tan(k\theta)$  despite the hint in the given answer that the solution involved logarithms. As the answer has been given, candidates were expected to demonstrate clear use of both limits, and to give a clear demonstration of how their solution simplifies to the given answer. The lower limit of the integral often appeared to have been ignored, and several candidates struggled to deal with the logarithms correctly.

#### Question 6

- (i) Most candidates should have been able to score at least one mark for a correct sketch of the line  $y = \frac{1}{3}x + 1$ , however often they did not pass through  $(0,1)$  or did not pass near  $(3,2)$ . Many candidates were clearly not familiar with the graph of  $\operatorname{cosec} \theta$ .

Although the question asks candidates to show that the equation has one root in the interval, few candidates made any mention of the point of intersection of the two graphs.

- (ii) Well prepared candidates were familiar with the expectation of this part of the question, they substituted values correctly into an appropriate function and drew an appropriate conclusion from the results. Candidates are expected to evaluate their function – it is not sufficient to claim one positive answer and one negative answer with no supporting evidence. Having obtained their values, candidates are expected to give some indication of how these demonstrate that there is a root in the interval. A small number of candidates were clearly working with their calculator set in degrees mode and gained no credit. Several candidates did not know how to evaluate the function  $\operatorname{cosec}\left(\frac{1}{2}x\right)$ , with  $2\cos x$  and  $\frac{1}{2\sin x}$  being seen as alternatives.

- (iii) There were some concise solutions to this question, working in both directions. There were several candidates who clearly did not understand the notation  $\sin^{-1}(\theta)$ , the most common error being to interpret it as  $\frac{1}{\sin \theta}$ . Some candidates offered solutions involving  $\frac{1}{\sin} \theta$ , which has no meaning. There were several candidates who struggled with the algebra, particularly among those who tackled the problem in reverse – the error  $\frac{3}{x+3} = \frac{3}{x} + \frac{3}{3}$  was common.

Some candidates did not see the distinction between **Part (ii)** and **Part (iii)** and attempted to solve **Part (iii)** by substituting values. It was quite common to see the answer to **Part (iv)** appearing as the answer to **Part (iii)**.

- (iv) Most candidates made a good attempt to apply the iterative formula correctly. Some stopped before they had reached two successive values rounding correctly to the required accuracy. Many candidates did not present their work or solution to the required level of accuracy. Candidates should read the question carefully to determine the accuracy required, and ensure that they understand how to round an answer to the required number of decimal places. Candidates working in degrees often did not recognise their error when their values were clearly not converging to a root in the given interval.

1.471

### Question 7

- (i) Many candidates applied the product rule and the chain rule correctly to obtain the correct derivative. There were some sign errors at this stage, but the most common error was incorrect differentiation of  $e^{\frac{1}{2}x}$ , obtaining either  $e^{\frac{1}{2}x}$ ,  $2e^{\frac{1}{2}x}$  or occasionally  $\frac{1}{2}e^{-\frac{1}{2}x}$ . Many candidates realised that to find the  $x$ -coordinate of  $M$  they needed to equate their derivative to zero; the algebra that followed was often incorrect. The question asks for an exact value, so decimal solutions were not accepted. Candidates who gave exact answers to incorrect quadratic equations without showing any working gained no credit – working must be shown.
- (ii) Many candidates demonstrated a good understanding of the method of integration by parts. Although the outline of the method was usually clear to see, there were several errors with the signs and constants, particularly at the second stage of integration by parts. A clear demonstration of the use of both limits was expected. The question asks for an exact solution, so substitution of a decimal approximation for  $e$  was not appropriate.

(i)  $\sqrt{5} - 1$

(ii)  $24 - 8e$



### Question 8

- (i) Most candidates were successful in picking out the normal vector for at least one of the two planes, and they understood that for planes to be perpendicular then their normal vectors must be perpendicular. The question tells them that the planes are perpendicular, so candidates were expected to demonstrate the evaluation of their scalar product – it was not sufficient to do the mental arithmetic and claim that the answer was zero. A common error was to have the normal vector of the first plane as  $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ .
- (ii) Candidates tried a variety of approaches to this question, and many made good progress. Candidates treating the two equations as a pair of simultaneous equations often made sign errors in the course of a correct attempt to eliminate one variable. Despite this, several candidates got as far as  $z = 5 - 4x$  and  $y = 7 - 7x$ . The next step, completing the solution, proved to be more difficult.

Calculating the vector product of the normal vectors of the two planes was a popular choice; this approach was sometimes beset with sign errors, either through the incorrect sign on the middle term or thought misapplication of the method with ' $ad + bc$ ' rather than ' $ad - bc$ '.

The question asks candidates to find the vector equation of the plane, so the answer should be of the form  $\mathbf{r} = \dots$

$$\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$$

### Question 9

- (a) Many candidates made a good start to this question, with a correct application of the quadratic formula, and despite several arithmetic errors in the course of the working most candidates gained at least the three method marks. Despite many arithmetic errors in evaluating the discriminant, most candidates did demonstrate correct use of  $i^2 = -1$ . Some candidates got as far as  $w = \frac{1}{2i+1}$  and  $w = -\frac{5}{2i+1}$  and stopped – they did not go on to express their answers in the required form. Although the method of multiplying top and bottom by the conjugate of the denominator was widely understood, it was not uncommon to see answers such as  $w = \frac{1}{2i} + 1$ .

A small number of candidates started by multiplying the whole equation by  $1 - 2i$  and were rewarded with a simpler application of the quadratic formula (or even straight forward factors).

It is often possible to solve this type of question by substituting  $w = x + iy$  and forming equations for the real coefficients and imaginary coefficients. It is possible in this case, although the pair of simultaneous equations that result are not straight forward to solve – none of the candidates who attempted this method got further than the pair of simultaneous equations.

The use of a calculator was ruled out in this question, so full working was expected at each stage.

- (b) Although the use of graph paper was not expected for the Argand diagram, it did make a difference in this question because it usually led to candidates using equal (and even) scales on both axes of their diagram. Many candidates recognised that the first inequality described a circular disc, although the diagrams they drew did not allow them to demonstrate this; if their diagram does not have equal scales then they should be drawing an ellipse. Many of the 'circles' did have the correct centre, but the radius was not always 2 in all directions. The second inequality describes two half lines which should start at the origin; it was a common error to see them starting at the centre of the circle. Although many candidates clearly had a good idea of what the inequalities described, their diagrams did not reflect this.

(a)  $\frac{1}{5} - \frac{2}{5}i$ ,  $-1 + 2i$

**Question 10**

- (i) A small number of candidates did not realise that the question required the use of calculus and set about trying to find  $k$  by substituting values into the differential equation. However, most candidates were successful in separating the variables in the differential equation, and went on to complete the integration with respect to  $t$  correctly. Many candidates did not realise that  $\int \frac{1}{x(4-x)} dx$  requires the use of partial fractions. Although a few candidates used substitution or found equivalent alternative routes, many used the incorrect answer  $\frac{1}{4-2x} \ln(4-x^2)$ .

Those candidates who integrated correctly were able to go on to evaluate the two constants involved. Many reached a correct expression for  $k$ , for example  $k = \frac{1}{8} \ln 9$  but did not go on to derive the given answer. The question asks candidates to solve the differential equation and find  $k$ . Several candidates focussed on the target of the value for  $k$  and never actually stated a solution for the differential equation.

- (ii) Only a small number of candidates offered a solution to this part of the question. Most of these did make the key step of translating 90% of the area into  $x = 3.6$ . Some of these candidates made errors because they confused  $(\ln 3)t$  with  $\ln(3t)$ .
- (ii) 4

# MATHEMATICS

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Paper 9709/32  
Paper 32

## General comments

The candidates found aspects of this paper quite challenging. Whereas most candidates demonstrated knowledge and understanding of the majority of topics, all too frequently solutions went wrong due to algebraic and arithmetic errors. Many candidates scored well on **Question 2** (binomial expansion), **Question 3** (trigonometric equation), **Question 6(iv)** (application of an iterative formula) and **Question 8(i)** (perpendicular planes). The response to **Question 1** (algebra and indices) was particularly disappointing; candidates did not seem to pause and reflect before working through the simple algebraic processes required. Other questions that candidates found challenging were **Question 4** (implicit differentiation), **Question 5(ii)** (integration of  $\tan \theta$ ), and **Question 10(i)** (solution of a differential equation).

In **Question 5(i)**, **Question 5(ii)**, **Question 6(i)**, **Question 6(iii)**, **Question 8(i)** and **Question 10(i)** candidates were asked to demonstrate a given answer. It is important for candidates to realise that in these circumstances they need to show full reasoning to support their conclusions in order to score full marks. Similarly, when a question says '... the use of a calculator is not permitted' or asks candidates to 'find the exact value...' this is a clear message to candidates that an answer copied from a calculator will not be sufficient to score the marks available.

Candidates should read through the questions carefully and ensure that their solution answers the question. In this paper this was a particular issue in **Question 2**, **Question 6(i)**, and **Question 10(i)**.

Candidates preparing for this paper should be aware of how easy it is to lose marks through algebraic and arithmetic errors. In particular, many candidates demonstrate poor and inaccurate use of brackets. A disciplined approach to algebra and accurate use of the notation is an important part of mathematical communication. Similarly, candidates need to be aware of the difference between 'significant figures' and 'decimal places' and to be able to use these accurately.

The standard of presentation of the work was often very good, and the best candidates made their reasoning clear by showing full working. With the advent of the marking of scanned scripts, candidates should be aware that if they erase work and write over it then the result is barely legible for the examiner – it is far better to cross through the original and replace it. When candidates have used additional answer booklets it is helpful if these are attached in the correct order.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

### Comments on specific questions

#### Question 1

There were many concise and correct solutions to this question. The most common errors involved incorrect statements such as  $8 \times 3^x = 24^x$  and  $24^x - 3^x = 21^x$ . Some candidates are so accustomed to solving modular inequalities in **Question 1** that they set about squaring the given equation. Another popular tactic was to multiply top and bottom of the fraction by  $3^x + 2$ . Those with strong algebra were able to work through the additional work that this created. This question required an answer 'correct to 3 decimal places'; some candidates truncated their answer rather than rounding it, or gave only 2 decimal places in their answer.

0.860

#### Question 2

Most candidates demonstrated a good understanding of how to use the binomial expansion for a rational index. Many solutions started with a correct unsimplified expansion of  $(1 + 2x)^{\frac{3}{2}}$  as far as the term in  $x^2$ . Simplified expressions were often incorrect due to slips in the arithmetic in dealing with  $(2x)^2$ . Some candidates used powers of  $x$  rather than of  $2x$ . A few candidates also attempted to expand  $(2 - x)$ , however most of them did obtain  $(2 - x)$  as their answer. Many candidates lost marks in this question due to errors in their algebra and arithmetic when multiplying their expansion by  $(2 - x)$ .

There was a common misunderstanding that the question was only asking for the term in  $x^2$  rather than terms 'up to and including the term in  $x^2$ '.

$2 - 7x + 18x^2$

#### Question 3

Access to this question depended on knowing the definition of  $\sec \theta$  followed by correct use of  $\cos^2 \theta + \sin^2 \theta = 1$ . Most candidates were successful in reaching a correct quadratic in  $\sin \theta$ . Although the quadratic was straight forward to factorise, several candidates preferred to use the quadratic formula. Candidates should be aware that if they do not state the formula correctly and make an error in applying it then this will be regarded as a method error. Many candidates did reach the correct values of  $\sin \theta$ . However, after extensive working they often rejected or did not consider the only solution within the required range.

$-41.8^\circ$

#### Question 4

Many candidates found this question challenging. Some candidates tried to avoid implicit differentiation through attempting to rearrange the equation before differentiating, however this was not possible in this case.

The most common approach was to multiply out the brackets and differentiate the two terms separately. Some errors crept in at this stage with not all candidates reaching  $x^2y - 6xy^2$  and those who did, often had a sign error when combining the derivatives of the two terms. Many candidates differentiated the constant term  $9a^3$  to obtain  $27a^2$  making further progress impossible. Only a small number of candidates substituted  $\frac{dy}{dx} = 0$  before making  $\frac{dy}{dx}$  the subject of an equation. Many got to an expression for  $\frac{dy}{dx}$  and went no further. Some thought that for the tangent to be parallel to the axis then either  $x = 0$  or  $y = 0$ . Only a minority of the candidates who reached a correct expression in  $x$  and  $y$  explicitly rejected the possibility of  $y = 0$ .

$$(-3a, -a)$$

#### Question 5

- (i) Many candidates made good progress in tackling the identity, although several took very circuitous routes. The most common errors involved losing the denominator when combining the two terms on the left hand side, and sign errors in removing brackets (which were often omitted). Some candidates appeared to confuse  $\tan 2\theta - \tan \theta$  with  $\tan(2\theta - \theta)$ .
- (ii) Many candidates did not see the link between the two parts of this question. Of those who did, the majority did not recognise the standard integral of  $\tan(k\theta)$  despite the hint in the given answer that the solution involved logarithms. As the answer has been given, candidates were expected to demonstrate clear use of both limits, and to give a clear demonstration of how their solution simplifies to the given answer. The lower limit of the integral often appeared to have been ignored, and several candidates struggled to deal with the logarithms correctly.

#### Question 6

- (i) Most candidates should have been able to score at least one mark for a correct sketch of the line  $y = \frac{1}{3}x + 1$ , however often they did not pass through  $(0,1)$  or did not pass near  $(3,2)$ . Many candidates were clearly not familiar with the graph of  $\operatorname{cosec} \theta$ .

Although the question asks candidates to show that the equation has one root in the interval, few candidates made any mention of the point of intersection of the two graphs.

- (ii) Well prepared candidates were familiar with the expectation of this part of the question, they substituted values correctly into an appropriate function and drew an appropriate conclusion from the results. Candidates are expected to evaluate their function – it is not sufficient to claim one positive answer and one negative answer with no supporting evidence. Having obtained their values, candidates are expected to give some indication of how these demonstrate that there is a root in the interval. A small number of candidates were clearly working with their calculator set in degrees mode and gained no credit. Several candidates did not know how to evaluate the function  $\operatorname{cosec}\left(\frac{1}{2}x\right)$ , with  $2\cos x$  and  $\frac{1}{2\sin x}$  being seen as alternatives.

- (iii) There were some concise solutions to this question, working in both directions. There were several candidates who clearly did not understand the notation  $\sin^{-1}(\theta)$ , the most common error being to interpret it as  $\frac{1}{\sin \theta}$ . Some candidates offered solutions involving  $\frac{1}{\sin} \theta$ , which has no meaning. There were several candidates who struggled with the algebra, particularly among those who tackled the problem in reverse – the error  $\frac{3}{x+3} = \frac{3}{x} + \frac{3}{3}$  was common.

Some candidates did not see the distinction between **Part (ii)** and **Part (iii)** and attempted to solve **Part (iii)** by substituting values. It was quite common to see the answer to **Part (iv)** appearing as the answer to **Part (iii)**.

- (iv) Most candidates made a good attempt to apply the iterative formula correctly. Some stopped before they had reached two successive values rounding correctly to the required accuracy. Many candidates did not present their work or solution to the required level of accuracy. Candidates should read the question carefully to determine the accuracy required, and ensure that they understand how to round an answer to the required number of decimal places. Candidates working in degrees often did not recognise their error when their values were clearly not converging to a root in the given interval.

1.471

### Question 7

- (i) Many candidates applied the product rule and the chain rule correctly to obtain the correct derivative. There were some sign errors at this stage, but the most common error was incorrect differentiation of  $e^{\frac{1}{2}x}$ , obtaining either  $e^{\frac{1}{2}x}$ ,  $2e^{\frac{1}{2}x}$  or occasionally  $\frac{1}{2}e^{-\frac{1}{2}x}$ . Many candidates realised that to find the  $x$ -coordinate of  $M$  they needed to equate their derivative to zero; the algebra that followed was often incorrect. The question asks for an exact value, so decimal solutions were not accepted. Candidates who gave exact answers to incorrect quadratic equations without showing any working gained no credit – working must be shown.
- (ii) Many candidates demonstrated a good understanding of the method of integration by parts. Although the outline of the method was usually clear to see, there were several errors with the signs and constants, particularly at the second stage of integration by parts. A clear demonstration of the use of both limits was expected. The question asks for an exact solution, so substitution of a decimal approximation for  $e$  was not appropriate.

(i)  $\sqrt{5} - 1$

(ii)  $24 - 8e$

### Question 8

- (i) Most candidates were successful in picking out the normal vector for at least one of the two planes, and they understood that for planes to be perpendicular then their normal vectors must be perpendicular. The question tells them that the planes are perpendicular, so candidates were expected to demonstrate the evaluation of their scalar product – it was not sufficient to do the mental arithmetic and claim that the answer was zero. A common error was to have the normal vector of the first plane as  $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ .
- (ii) Candidates tried a variety of approaches to this question, and many made good progress. Candidates treating the two equations as a pair of simultaneous equations often made sign errors in the course of a correct attempt to eliminate one variable. Despite this, several candidates got as far as  $z = 5 - 4x$  and  $y = 7 - 7x$ . The next step, completing the solution, proved to be more difficult.

Calculating the vector product of the normal vectors of the two planes was a popular choice; this approach was sometimes beset with sign errors, either through the incorrect sign on the middle term or thought misapplication of the method with ' $ad + bc$ ' rather than ' $ad - bc$ '.

The question asks candidates to find the vector equation of the plane, so the answer should be of the form  $\mathbf{r} = \dots$

$$\mathbf{r} = 7\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} - 7\mathbf{j} - 4\mathbf{k})$$

### Question 9

- (a) Many candidates made a good start to this question, with a correct application of the quadratic formula, and despite several arithmetic errors in the course of the working most candidates gained at least the three method marks. Despite many arithmetic errors in evaluating the discriminant, most candidates did demonstrate correct use of  $i^2 = -1$ . Some candidates got as far as  $w = \frac{1}{2i+1}$  and  $w = -\frac{5}{2i+1}$  and stopped – they did not go on to express their answers in the required form. Although the method of multiplying top and bottom by the conjugate of the denominator was widely understood, it was not uncommon to see answers such as  $w = \frac{1}{2i} + 1$ .

A small number of candidates started by multiplying the whole equation by  $1 - 2i$  and were rewarded with a simpler application of the quadratic formula (or even straight forward factors).

It is often possible to solve this type of question by substituting  $w = x + iy$  and forming equations for the real coefficients and imaginary coefficients. It is possible in this case, although the pair of simultaneous equations that result are not straight forward to solve – none of the candidates who attempted this method got further than the pair of simultaneous equations.

The use of a calculator was ruled out in this question, so full working was expected at each stage.

- (b) Although the use of graph paper was not expected for the Argand diagram, it did make a difference in this question because it usually led to candidates using equal (and even) scales on both axes of their diagram. Many candidates recognised that the first inequality described a circular disc, although the diagrams they drew did not allow them to demonstrate this; if their diagram does not have equal scales then they should be drawing an ellipse. Many of the 'circles' did have the correct centre, but the radius was not always 2 in all directions. The second inequality describes two half lines which should start at the origin; it was a common error to see them starting at the centre of the circle. Although many candidates clearly had a good idea of what the inequalities described, their diagrams did not reflect this.

(a)  $\frac{1}{5} - \frac{2}{5}i$ ,  $-1 + 2i$



**Question 10**

- (i) A small number of candidates did not realise that the question required the use of calculus and set about trying to find  $k$  by substituting values into the differential equation. However, most candidates were successful in separating the variables in the differential equation, and went on to complete the integration with respect to  $t$  correctly. Many candidates did not realise that  $\int \frac{1}{x(4-x)} dx$  requires the use of partial fractions. Although a few candidates used substitution or found equivalent alternative routes, many used the incorrect answer  $\frac{1}{4-2x} \ln(4-x^2)$ .

Those candidates who integrated correctly were able to go on to evaluate the two constants involved. Many reached a correct expression for  $k$ , for example  $k = \frac{1}{8} \ln 9$  but did not go on to derive the given answer. The question asks candidates to solve the differential equation and find  $k$ . Several candidates focussed on the target of the value for  $k$  and never actually stated a solution for the differential equation.

- (ii) Only a small number of candidates offered a solution to this part of the question. Most of these did make the key step of translating 90% of the area into  $x = 3.6$ . Some of these candidates made errors because they confused  $(\ln 3)t$  with  $\ln(3t)$ .
- (ii) 4

# MATHEMATICS

Paper 9709/33  
Paper 33

## General comments

The candidates found aspects of this paper quite challenging. In particular **Question 4(i)**, **Question 6(ii)** and **Question 9(i)**, where candidates failed to read the whole question or think of their approach prior to commencing their solution. This was clearly in evidence as a large number of candidates required extra booklets with the first 12 page booklet filled with numerous pages of crossed out work. In most cases these were virtually full solutions, with three or more attempts at the same section. In total this crossed out work was probably equivalent to the candidate having to solve an extra five or six questions. Such a demand on time often leads to careless mistakes elsewhere in the paper.

Most candidates appeared to have taken on board the comments in recent 9709/33 reports regarding showing their full working when they're in a given answer, clear substitution of limits when evaluating a definite integral, using the quadratic formula when solving a quadratic equation and what is meant by an exact answer.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on individual questions.

### Question 1

The solution to this question required the application of the quotient law of logarithms, followed by the removal of the logarithm term, finishing with some basic algebra to express  $y$  in terms of  $z$ . Most candidates quickly produced the correct solution, although a number struggled with the algebra at the end. Notation or understanding was weak in a few cases as  $\ln((y+2)/(y+1))$  was often presented as  $\ln(y+2)/\ln(y+1)$ . Another too common an error that appeared was the denominator in the single  $\ln$  expression, written immediately following the correct transcript of the expression for  $z$  onto the candidate's script, as  $(y-1)$  instead of  $(y+1)$ .

$$\text{Answer: } y = \frac{2 - e^z}{e^z - 1}$$

### Question 2

This solution required the use of the derivative rule for a quotient or product expression, followed by the application of Pythagoras, finishing with a clear sound argument. The differentiation was usually performed correctly, although within the numerator of the quotient formula some candidates had either the incorrect signs of both terms or a positive sign instead of a negative one. Several candidates omitted brackets and as a result any accuracy marks were soon no longer available. While most candidates could make the Pythagoras application, arithmetical errors sometimes prevented their simplified expression being in the form required to proceed with the final part of the question. Candidates should realise that while one can disprove something with a counter example one cannot prove something by an example. Hence candidates who evaluated the expression at one or more values of  $x$ , usually  $\pm \pi$ , or  $\pi/2$ , failed to justify the statement. Much

better were candidates who produced a sketch or stated  $-1 < \cos x < 1$ , so  $0 < 1 + \cos x$  or  $0 < 1 + \cos x < 2$ . Unfortunately then many of these candidates undid all their previously excellent work by making an incorrect statement, such as  $0 < 1/(1 + \cos x) < \frac{1}{2}$ , instead of  $\frac{1}{2} < 1/(1 + \cos x)$ .

Answer:  $y = \frac{1}{1 + \cos x}$

### Question 3

Most candidates were quickly successful with a relatively brief solution. It was rare to see an incorrect formula for  $\tan 2\theta$ , and only occasionally was an incorrect quadratic equation produced for  $\tan \theta$ . Candidates appeared to have little problem deciding the required quadrant for their angle, although for some reason due to incorrect reasoning about  $\cot 2\theta$  at  $\theta = 135^\circ$ , the value of  $\theta = 135^\circ$  was rejected. Working in terms of radians was virtually never seen.

Answers:  $18.4^\circ$ ,  $135^\circ$

### Question 4

- (i) This question appeared to be the first to cause problems. Had candidates read the whole question, that is (i) and (ii), before commencing they would have realised that finding roots was related to (ii) and not (i). Hence the method of determining roots was something that should have been avoided in (i). While it is possible, it does if performed correctly lead to a complex root to substitute into  $p(x)$ , followed by the taking of real and imaginary parts. Many candidates adopted this approach but instead of obtaining complex roots obtained the incorrect real roots  $x = -1$  and  $x = 2$ , or with other signs, for the solution of a very simple quadratic equation.

The candidates who decided upon the sensible approach of either long division or considering another quadratic polynomial of the form  $(Ax^2 + Bx + C)$  were more successful. The latter approach was nearly always the most successful of these two since it avoided the necessity of multiply by a term containing the parameter  $a$ , as opposed to a number which is the norm. Unfortunately many candidates who opted for the long division approach became confused as to whether they were subtracting or adding rows, so few candidates finished with the correct remainder. The problem then arose as to what to do with this remainder, although some candidates correctly set the coefficients of the linear term and the constant both to zero, or substituted 2 values of  $x$  and solved the resulting equations, many were confused and decided to set it either to the linear part of  $p(x)$  or to the coefficients of the linear terms in their new quadratic polynomial. Hence, candidates appeared to jump from one approach to another or to embarked on several full solutions using the same approach but including different errors.

Answers:  $a = 1$   $b = -6$

- (ii) Without the correct quadratic polynomial the real roots were obviously not possible. However, candidates must do more than state, could simply be a guess, that roots to the quadratic expression  $(x^2 - x + 2)$  are complex, they must either find these complex roots or at least show that the discriminant is negative in order to justify their statement.

Answers: real roots  $\frac{1}{2}$  and  $-\frac{3}{2}$  imaginary roots  $(1 \pm i\sqrt{7})/2$  or discriminant =  $-7$

### Question 5

- (i) Unfortunately many candidates didn't really know what to do in this question and never had  $dy/dx$  in their expression. Many knew that  $m$  was  $xy/2$  but were unable to continue.

Answer:  $\frac{dy}{dx} = \frac{xy}{2}$

- (ii) Candidates needed (i) correct to make any real progress with this section, and virtually correct for the method mark for separating variables correctly or the method mark for substituting  $(0, 2)$  to find the constant of integration. If (i) was correct then the candidate usually scored all the marks but for

final answer mark. This was often lost due to an incorrect procedure when removing logarithms. That is when exponentiating  $x^2/4 + \ln 2$  often incorrectly expressed as  $\exp(x^2/4) + 2$ .

Answer:  $y = 2e^{\frac{1}{4}x^2}$

- (iii) For a correct sketch a correct solution was required in (ii). In addition it was necessary to state the coordinates where the graph intersected any axes, in this case (0, 2). Although the question states  $x > 0$ , if a candidate decides to show the solution elsewhere, as in the region  $x < 0$ , then it must be correct, exactly as in **Question 2** when the candidate produced incorrect additional details. Of those candidates that were in a position to produce a correct graph in (iii) probably 50 per cent had  $y \rightarrow 0$  as  $x \rightarrow -\infty$  instead of a graph symmetrical about the  $y$  axis.

### Question 6

- (i) Most candidates successfully used the given substitution to convert the integral from a function of  $x$  to a function of  $u$ . However, when undertaking this with a given answer it is necessary to perform every step in detail. For example, (a) the clear conversion of the limits from 1 - 4 to 1 - 2 outside of the integral itself, since they are already stated there, (b) the clear cancellation of  $u$  from the numerator and denominator following its factorisation from both, (c) finding  $\frac{dx}{du} = 2u$  or an equivalent correct statement, etc.
- (ii) Like **Question 4(ii)** most candidates had numerous attempts at trying to establish the given answer. The fact that it was a given answer meant that every time the candidate failed to obtain this result they deleted their attempt and tried again. Even among those candidates trying partial fractions from the start most failed to recognise that this was an improper rational fraction and hence needing dividing out at some stage, preferably at the start. The alternative approach was to introduce the substitution  $z = u + 1$ , with either approach producing two standard integrals following a few lines of working.

### Question 7

- (i) While the modulus was usually correct, often the argument was given as  $60^\circ$  or  $\pi/3$ .

Answers: Modulus  $2\sqrt{2}$  Argument  $-\pi/3$  or  $-60^\circ$

- (ii) Likewise candidates found  $z + 2z^*$  straight forward, but often struggle with  $\frac{z^*}{iz}$ . The problems were two fold, either failing to express the denominator as a single complex number in order to establish the required complex conjugate to multiply the numerator and denominator by, or poor multiplication and addition of the combinations of the surds  $\sqrt{2}$  and  $\sqrt{6}$ .

Answer:  $3\sqrt{2} + \sqrt{6}i$   $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$

- (iii) The points  $A$  and  $B$  were usually shown correctly and candidates who failed to score this mark did so for one of the following reasons, scales on real and imaginary axes different, as equal scales is an essential part of an Argand diagram, non uniform scales or length of  $OA$  not approximately the same as that of  $OB$  (should have been identical). Proving the final point followed immediately from the argument of (ii) (b), simply from the simplification of  $\arctan\left(\frac{1/2}{\sqrt{3}/2}\right)$  to  $\arctan(1/\sqrt{3})$ .

Candidates who opted to commence with the difference of the arguments of  $z^*$  and  $iz$  needed to ensure that these expressions were reduced to the inverse of tangent quantities that they knew, namely those of  $1/\sqrt{3}$  and  $\sqrt{3}$ , and not left in more complicated forms. However, it was possible to use either the scalar product or the cosine formula, as all the calculations using the real and imaginary parts of the complex numbers involved, required nothing more than mental arithmetic.

### Question 8

- (i) Most candidates produced their best work in this section, with completely correct answers common. A few candidates made arithmetical errors, while others started from an incorrect form, opting for  $C/(x^2 + 4)$  instead of  $(Bx + C)/(x^2 + 4)$  as one of their partial fractions. Unfortunately, the omission of brackets from the term  $(Bx + C)$  when multiplying throughout by  $(x + 2)(x^2 + 4)$  presented a serious problem for those candidates making this error. The fact that by choosing  $x = 2$  and  $x = 0$  they obtained the correct values for the constants  $A$  and  $C$  was simply fortuitous as the resulting equations are incorrect for all other values of  $x$  and if solved with such values then all the constants would be incorrect. Basically the resulting equations are incorrect, and how many constants one determines correctly depends on the values of  $x$  used and the order in which the equations are solved.

A few candidates produced a rather ingenious and unusual approach, that is they first found the constant  $A$  in the term  $A/(x + 2)$  and then moved this term to the other side of the expression and subtracted from  $f(x)$  using a common denominator. Hence with virtually no calculation other than cancelling the common factor  $(x + 2)$  in the numerator and denominator the coefficients in the term  $(Bx + C)/(x^2 + 4)$  were established. However, unfortunately some of these candidates commenced with their form for the partial fraction involving  $C/(x^2 + 4)$  which meant using this approach resulted in the incorrect final statement of  $(x - 1)/(x^2 + 4) = C/(x^2 + 4)$ . As above, the omission of the  $Bx$  has fortuitously produced what appears to be a correct answer, simply because one has solved for the coefficient  $A$  first. If instead one had solved for the coefficient  $C$  first, which is possible by choosing  $x = 2i$ , (which is perfectly valid) then  $C$  would have been an incorrect complex coefficient and everything else would have been incorrect.

Answer:  $\frac{2}{x+2} + \frac{x-1}{x^2+4}$

- (ii) Again many completely correct solutions were seen. Errors that did occur were mainly due to the extraction of the constants 2 and 4 being included in the numerators of the appropriate terms as opposed to their denominators. However, miscopying of signs from correct expressions in (i) into (ii) was not uncommon, so  $(x - 2)^{-1}$ ,  $(x + 1)$  and  $(x^2 - 4)^{-1}$  all seen. Another error produced by some candidates was that when expanding  $(x^2 + 4)^{-1}$  and  $(x + 2)^{-1}$  it is also deemed necessary to have a negative sign outside the bracket, for example  $-2^{-1}(1 - x/2 + \dots)$ .

Answer:  $\frac{3}{4} - \frac{1}{4}x + \frac{5}{16}x^2$

### Question 9

- (i) Most candidates knew that they had to equate the two  $y$  values. Also many realised that differentiation was required, but not that equating the two values of  $\frac{dy}{dx}$  was essential in order to make any real progress. Many simply equated the derivative of the oscillatory function to zero, since they clearly didn't understand the meaning of "touch" in this question. While the derivative of  $y = x \cos x$  was usually correct, few could successfully differentiate  $y = k/x$ . Too many candidates wasted time by having no method but simply trying to equate different expressions in order to achieve the given equation.

Mostly correct, but some candidates failed to show convergence, while others showed convergence but then rounded to the incorrect value, for example 1.079 or 1.08.

A few candidates used degrees throughout (ii) and (iii) and since by the very nature of the question this is a basic error and produced nothing resembling a root between 0 and  $\pi/2$ .

Answer: 1.077

- (ii) Usually correct, but some candidates appeared not to be able to acquire an expression in this section for  $k$  in terms of  $a$  despite having clearly stated it in (ii). A few candidates gave the answer to three decimal places, namely 0.550, instead of the requested two decimal places.

Answer: 0.55

### Question 10

- (i) A diagram showing the line  $l$  and a vector to a point on this line a distance  $\sqrt{10}$  from the origin would have helped avoid a lot of confusion on this question. Most candidates that established the correct equation usually solved the quadratic equation by factorisation and produced the two correct vectors (coordinate expressions were also credited with full marks).

Answers:  $-i + 3j$  and  $\frac{7}{3}i + \frac{4}{3}j + \frac{5}{3}k$

Again many candidates realised that they needed the normal vector to plane  $p$  but then believed that the other vector in the calculation was  $i + 2j + k$ , not the direction vector  $2i - j + k$ . However, most knew that they needed to take the scalar product, obtain the moduli of the appropriate vectors, then divide the scalar product by the product of the moduli and equate the result to the cosine of the angle between these two vectors. It was at this stage that candidates experienced problems as the question states the value of the sine of the angle between the line  $l$  and the plane  $p$ , namely the required cosine of the angle between the plane  $p$  and direction vector. Again a diagram would have avoided the problems of candidates substituting  $\sqrt{5}/3$  for the required cosine instead of  $2/3$ . However, much worse saw candidates misunderstanding the meaning of  $\sin^{-1}(2/3)$  and substituting actual degrees instead of  $2/3$ . The detail working for the scalar product and the moduli were clearly shown, as stressed in previous reports, since many of these quickly were miscopied or processed incorrectly, so  $2a - 1 + 1$  became  $2a + 1$ ,  $\sqrt{(a^2 + 1 + 1)}$  became  $\sqrt{(a^2 + 1)}$ , etc. Unfortunately, many candidates who established the correct equation then produced some very poor algebra, such as squaring only one side of the equation, replacing  $\sqrt{(a^2 + 2)}$  by  $a + \sqrt{2}$ , or other such elementary incorrect algebraic statements. After achieving the correct answer it was disappointing to see some candidates for some unknown reason decide to reject their negative solution.

Answers:  $\pm 2$

# MATHEMATICS

Paper 9709/41  
Paper 41

## General comments

In general candidates performed well on the early questions on the paper but the final two questions were not so well done.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black/dark blue pen.

Some candidates lost marks due to not giving answers to 3sf as requested and also due to prematurely approximating within their calculations leading to the final answer. Students should be reminded that if an answer is required to 3sf then their working should be performed to at least 4sf.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct given answer unless this value is used such as in this paper in **Question 2(i)**.

## Comments on specific questions

### Question 1

Most candidates performed well on this question. It was necessary to apply Newton's second law to both particles. Particle  $Q$  has two forces acting on it, namely its weight and the tension in the string. The horizontal motion of particle  $P$  is only affected by the tension. However, some candidates also wrongly included the weight when considering the horizontal motion of particle  $P$ . Once the two equations of motion were stated, these two simultaneous equations could be solved for the required tension and acceleration.

*Answers:* The tension in the string is 2.4 N The acceleration of  $Q$  is  $4 \text{ ms}^{-2}$

### Question 2

- (i) Again most candidates performed well on this part of the question. Since the forces acting on the particle are constant, the acceleration can be found using the constant acceleration formulae. Once this has been found, applying Newton's second law down the plane enables the frictional force to be found. Although it is possible to solve the problem using the work/energy principle, most candidates applied Newton's law.

*Answers:* The acceleration of the particle is  $0.4 \text{ ms}^{-2}$  Frictional force = 0.302 N (Given)

- (ii) This part merely involved applying the formula  $F = \mu R$  to determine the coefficient of friction. As  $F$  is given in **part (i)** only the normal reaction  $R$  is to be found. An error that was seen was taking  $R$  as merely the weight of the particle rather than the correct component form which was  $R = 0.1g \cos 20$ . Substitution of the values of  $F$  and  $R$  into the equation  $\mu = F \div R$  gives the required value of the coefficient of friction.

*Answer:* The coefficient of friction between the particle and the plane is 0.321



### Question 3

- (i) There were essentially two different approaches made to solving this problem. As the velocity was given when the particle is at a height of 0.5m, one method of approach is to take this as the initial position and the constant acceleration equation  $v^2 = u^2 + 2as$  used with  $u = 6$ ,  $v = 0$  and  $a = -10$  in order to find  $s$ , the extra height the particle would travel before coming to rest. Once this had been found then  $s + 0.5$  gives the greatest height above  $O$ . Alternatively the value of the initial velocity at the ground could be found using the equation  $v^2 = u^2 + 2as$  with  $v = 6$ ,  $a = -10$  and  $s = 0.5$  to find  $u$  as  $\sqrt{46}$ . The total height  $h$  could then be found using  $v^2 = u^2 + 2as$  with  $u = \sqrt{46}$ ,  $v = 0$ ,  $a = -10$  and  $s = h$  is the greatest height above  $O$ .

*Answer:* The greatest height reached by the particle above  $O$  is 2.3 m

- (ii) Once the greatest height above  $O$  has been found the required time can again be determined in a variety of ways. Most candidates used the equation  $s = ut + \frac{1}{2}at^2$  with their value of the greatest height as  $s$ ,  $u = 0$  and  $a = 10$  to find  $t$ . This gives the time to fall from the greatest height back to  $O$ . The total time involves doubling this value. Alternatively the same equation could be used with the initial velocity at the ground taken as  $u = \sqrt{46}$ ,  $a = -10$  and  $s = 0$  which gives the total time to return to  $O$ . Most candidates used one or other of these methods.

*Answer:* The time after projection at which the particle returns to  $O$  is 1.36 seconds

### Question 4

This question was well done by the majority of candidates. The most common method was to resolve forces horizontally and vertically giving two equations involving  $F$  and  $\alpha$ . An error made by some candidates was merely to resolve forces and not produce an equation which made it difficult for them to proceed. One method of solution is to eliminate  $\alpha$  by writing the two equations with  $\cos \alpha$  as the subject of one,  $\sin \alpha$  as subject of the other and then by using  $\cos^2 \alpha + \sin^2 \alpha = 1$  this produced a value of  $F$ . Substitution of this value of  $F$  enabled  $\alpha$  to be found. Alternatively  $F$  could be eliminated giving an equation  $\tan \alpha = \sqrt{3/5}$  which can be solved for  $\alpha$ . Substitution into one of the original equations then gives  $F$ . In either of these methods some candidates struggled with the trigonometry involved. Most questions involving force systems such as this will require similar approaches and a good ability in trigonometric manipulation is vital.

*Answers:*  $F = 5.67$  and  $\alpha = 19.1$

### Question 5

- (i) Most candidates did not have a problem with this part. The required acceleration is found directly from the graph as the gradient of the line joining  $(0,0)$  to  $(10,5)$  on the graph

*Answer:* The acceleration of the cyclist over the first ten seconds is  $0.5 \text{ ms}^{-2}$

- (ii) Again most candidates made a successful attempt at this part. In order to find the required distance travelled in 90 seconds the area under the graph from  $t = 0$  to  $t = 90$  has to be found. This involved evaluating and adding together all of the parts that made up the area under the graph. This could be performed by using areas of a combination of triangles, rectangles and trapezia. However, candidates must be extremely careful to show all of their working as is always the case when an answer is given in the question. Most then successfully found the correct value of  $V$

*Answer:*  $V = 7$

- (iii) Candidates found this part of the question more challenging. There are two distinct methods of approach. One method is to apply Newton's second law to the motion down the slope. The length of the slope is found from the graph to be 60 m. The acceleration of the cyclist can be found by using the constant acceleration equation  $v = u + at$  with  $v = 7$ ,  $u = 5$  and  $t = 10$  giving  $a = 0.2$ . If the slope has angle  $\alpha$  then Newton's law applied down the slope leads to  $\sin \alpha = 0.045$  and the required vertical distance is  $60 \sin \alpha$ . An alternative approach uses the work/energy equation stating that the initial PE = increase in KE + work done against friction. Both methods were seen by candidates but often some of the terms involved were omitted, such as not involving KE when using work/energy equation or trigonometric errors in the Newton approach.

*Answer:* The vertical distance which the cyclist descends is 2.7 m

### Question 6

- (i) This part involved using the definition of work done. Some candidates merely multiplied the force 50 N by the distance 20 m but the definition of work done is "if an object moves a distance  $s$  along a line under the action of a force of magnitude  $F$  at an angle  $\theta$  to the line then the work done by the force is  $Fs \cos \theta$ ". The question only refers to the pulling force but some candidates also involved the frictional force which should not be included here. Hence in this case the work done is  $50 \times 20 \cos 10$

*Answer:* Work done by the pulling force is 985 J

- (ii) In this part, marks can only be scored if an energy method is used. This involves using the fact that the work done by the pulling force as found in **part (i)** = the increase in KE + work done against friction. From this equation the required value of  $v$  can be found. Some candidates also tried to include potential energy in their equation but there is no change in PE as the block does not change its height in this part of the question.

*Answer:* The speed of the block after it has moved 20 m is  $5.55 \text{ ms}^{-1}$

- (iii) This part involves using the definition of power as  $P = Fv$  and since the question asks for the greatest power then  $F = 50 \cos 10$  and  $v = 5.55$  since this is the greatest speed. Most candidates attempted to use the correct equation but sometimes using a force of 50 N but the force must be in the same direction as  $v$ .

*Answer:* The greatest power exerted by the 50 N force is 273 W

- (iv) Again there are two possible approaches and candidates generally used one or other of these. One method is to use the work/energy equation as the block moves up the slope and comes to rest, stating that PE gain + Work Done against Friction = Work done by 50 N force + Initial KE and this enables the distance up the plane to be found. By using  $s = \frac{1}{2}(u + v)t$  the required time can be found. Alternatively Newton's second law can be used on the slope involving the following forces: the component of the 50 N force, the component of the weight of the block and the friction force. This will give the acceleration  $a$  (negative value). Finally using  $v = u + at$  gives the value of  $t$ . Errors were often seen in this question. Most frequently this involved either omitting a term in the energy equation or missing out one of the forces in the Newton method.

*Answer:*  $t = 54.4$  seconds

### Question 7

- (i) This question involved either drawing a sketch of the acceleration for  $0 \leq t \leq 5$  or differentiating the expression for  $a$  over this range to find its maximum point. Some candidates were unsure as to how to deal with finding this maximum, some integrating instead of differentiating. Others wrongly attempted to use the constant acceleration formula. A sketch of the quadratic curve or the differentiation method showed that the maximum occurred at  $t = 2.5$ . Substitution into the expression for acceleration gave the required answer.

*Answer:* The maximum acceleration in the first five seconds is  $18.75 \text{ ms}^{-2}$

- (ii) The method for finding the distance involved integration of the given acceleration in order to find the velocity  $v$  and then a further integration to find the displacement  $s$ . Substitution of  $t = 5$  into this expression for displacement gave the required result. Those who used integration generally scored well here but some candidates again wrongly used the constant acceleration formulae to find  $s$  from the given expression for  $a$ .

*Answer:* The distance of the car from its starting point at  $t = 5$  is 156.25 m

- (iii) This part was found to be particularly difficult for candidates. As it was necessary to match up the velocities between the two periods at  $t = 5$  the value of the velocity at the end of the first 5 seconds had to be found by substitution as  $62.5 \text{ ms}^{-1}$ . The expression for acceleration beyond  $t = 5$  was found by many to be difficult to integrate in order to find the velocity with many making sign errors or not realising how to integrate an expression of the form  $A/t^2$ . If the integration was performed it was vital to include the constant of integration giving  $v = 625/t + C$  and  $C$  could be found by matching the velocity at  $t = 5$  giving  $v = 625/t - 62.5$ . Candidates who reached this stage found it straightforward to find the value of  $t = k$  at which the velocity was zero by solving  $0 = 625/k - 62.5$ . However, very few candidates reached this stage and obtained the correct value of  $k$ .

*Answer:*  $k = 10$

# MATHEMATICS

Paper 9709/42  
Paper 42

## General comments

The first five questions on the paper were generally well done by many candidates but many struggled with the final two questions. More details are given below.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

Some candidates lost marks due to not giving answers to 3sf as requested and also due to prematurely approximating within their calculations leading to the final answer. Students should be reminded that if an answer is required to 3sf then their working should be performed to at least 4sf.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct given answer unless this value is used such as in this paper in **Question 1(i)**

## Comments on specific questions

### Question 1

- (i) Most candidates attempted to apply Newton's second law to the particle in the direction of motion. This required candidates to find the acceleration of the particle from the given information. As the applied force is constant, the constant acceleration equations can be used to determine the acceleration. The forces acting on the particle are the component of the 10 N force in the direction of motion and the required frictional force. Most candidates correctly equated the combination of these forces to mass  $\times$  acceleration and achieved the given result. Some candidates misread the question, believing that the 15 degrees referred to a slope on which the particle was moving. Whenever a result is given in the question as it was here, candidates must show all working clearly in order to convincingly prove the result.

*Answer:* Magnitude of the frictional force is 8.96 N (Given)

- (ii) In order to find the required coefficient of friction, candidates needed to use the equation  $F = \mu R$ . Since  $F$  had been given in **part (i)** it was necessary to find  $R$ . Some candidates wrongly quoted  $R = 2g$ , forgetting that the 10 N force has a component in the vertical direction. The correct form for  $R$  is  $R = 2g - 10\sin 15$ . However, some candidates resolved incorrectly and found  $R$  as  $2g + 10\sin 15$ . When as in this case the value of  $F$  is given in **part (i)**, this value must be used in **part (ii)** whatever value the candidate finds in **part (i)**.

*Answer:* The coefficient of friction between the particle and the plane = 0.515

### Question 2

- (i) Almost all candidates performed well on this part of the question, differentiating to find the velocity  $v$  and then differentiating again to find the acceleration  $a$ . However, many made algebraic errors when solving the equation  $a = 0$  usually incorrect understanding of fractional powers with many finding  $t = \sqrt{5}$  instead of  $t = 5^2$ .

*Answers:* The acceleration of the particle is zero at  $t = 25$

- (ii) For those candidates who correctly found  $t = 25$  in **part (i)**, it was merely a case of substituting their value into the expressions for  $s$  and  $v$ . However, in spite of correct substitution, many did not believe their own working and rejected the minus sign which appears in both answers. Candidates who obtained an incorrect value for  $t$  in **part (i)** were able to score the method mark in **part (ii)**.

*Answers:* The displacement  $s = -2080$  m    The velocity  $v = -100$  ms<sup>-1</sup>

### Question 3

- (i) This part of the question was very well done by almost all candidates who added the two components of the forces along the direction of the river. A few candidates did not consider components and wrongly stated the answer as  $50 + 60 = 110$

*Answer:* The total force exerted by the two people in the direction of the river =  $60\cos 25 + 50\cos 15 = 103$  N

- (ii) Most candidates answered this part by first finding the component of the forces perpendicular to the direction of the river (12.4 N) and then using Pythagoras to find the resultant force and inverse trigonometry to find the required direction. Most candidates correctly found the value of the resultant force but although many found the angle correctly, they lost the final mark because they did not correctly explain the direction. Candidates would be advised to show answers to questions such as this in a diagram as it is difficult to explain the direction without a diagram.

Some candidates found the resultant force by using the cosine rule to combine the 50 N and 60 N forces. This is perfectly acceptable. The angle can be found using the sine rule. It is not advisable to attempt to solve the problem by using a scale drawing as it is then very difficult to achieve the required 3sf accuracy

*Answers:* The magnitude of the resultant force is 103 N.

This force acts in a direction  $6.9^\circ$  anticlockwise from the direction of the river.

### Question 4

- (i) The two most popular methods of solving this problem were using energy conservation or to use the constant acceleration formulae. Those who used initial KE + initial PE = final KE (using the bottom of the slope as the zero PE level) usually achieved the correct answer. However, some forgot to include the initial KE due to the starting speed of  $5$  ms<sup>-1</sup>. Another error that was seen was to use initial PE as  $mg \cdot 100$  rather than the correct form of  $mg \cdot 100\sin 20$ . Those who used constant acceleration formulae had to remember that the motion is along the slope and so the acceleration along the slope was required. Many candidates used  $a = g = 10$  rather than the correct value of  $a = g\sin 20 = 3.42$ . It was then necessary to use the formula  $v^2 = u^2 + 2as$  to determine the value of  $v$  at the bottom of the slope where  $s = 100$ . In this method some candidates forgot to include the initial velocity in their calculations.

*Answer:* The speed of the sledge at the bottom of the slope =  $26.6$  ms<sup>-1</sup>

- (ii) In this part it was necessary to use the work/energy equation as the work done against friction was given as 8500 J and this means that it cannot be assumed that the frictional force is constant. Many wrongly assumed that the frictional force was in fact 85 N and used constant acceleration formulae. Those who used the correct method had to use Initial KE + Initial PE = Final KE + 8500. Again many forgot to include the contribution of the initial KE.

*Answer:* The total mass of the girl and the sledge = 63.4 kg

### Question 5

Many candidates made errors in starting this question. It has to be considered as two separate cases, one where the 10 N force is acting and another where the 75 N force acts. In both cases it produces a balance between three forces. In the case of the 10 N force, the particle is about to slide down the plane and the equation of equilibrium is  $10 + F = mg\sin 30$  whereas when the 75 N force is applied the particle is about to slide up the plane and the equation is  $75 - F = mg\sin 30$  where the frictional force in both of these cases is  $F = \mu mg\cos 30$ . The two equations can then be solved for  $m$  and  $\mu$ . Some candidates did not use a balance between these three forces but wrongly included an acceleration term. Several candidates forgot to include the frictional force in the 10 N case and immediately found an incorrect value for  $m$ . Using the correct equations simple addition of these two equations gives the value for  $m$  and then substitution of this value into either equation gives  $\mu$ . However, many who found the two equations correctly then struggled to solve the two simultaneous equations for  $m$  and  $\mu$ .

*Answers:*  $m = 8.5$  The coefficient of friction is 0.442

### Question 6

- (i) As the system is travelling at constant speed the driving force exactly balances the total resistance to motion. Many candidates wrongly used 300 N as the driving force but since this force is applied to the whole system then it must include all resistances and should be  $300 \text{ N} + 100 \text{ N} = 400 \text{ N}$ . In order to find the power of the van's engine it was then necessary to use the formula  $P = Fv$  with  $F = 400$  and  $v = 25$ .

*Answer:* The power of the van's engine is  $P = 10\,000 \text{ W} = 10 \text{ kW}$

- (ii) In this part, only the trailer needs to be considered as the only forces acting on it are the tension in the cable and the resistance. These must balance and this immediately gives the value of the tension  $T$ .

*Answer:* The tension in the cable is 100 N

- (iii) Many candidates struggled with this part of the question. The application of Newton's second law separately to the van and to the trailer was needed. Either of these equations could be replaced by the equation applied to the system. These produce a pair of simultaneous equations involving the tension in the cable,  $T$ , and the acceleration,  $a$ . Many candidates forgot that there was an acceleration involved. As the driving force varies from 1000 N at the bottom of the hill to 1250 N at the time at which  $v = 20$ , it is not possible to use constant acceleration methods or the usual work/energy equations. The question only refers to what is happening at the instant that the speed is  $20 \text{ ms}^{-1}$ . Some candidates also forgot to include the effect of the component of the weight acting against the motion.

*Answer:* The new tension in the cable =  $1550/7 \text{ N} = 221 \text{ N}$

### Question 7

- (i) Almost all candidates performed well on this part. The best way to attempt this question was to draw a  $v-t$  diagram. Candidates either worked out the area of the triangle which described the motion over the first ten seconds and then added the area of the rectangle covering the motion from 10 seconds up to 40 seconds or alternatively they used the area of a suitable trapezium to evaluate the required distance. Use of constant acceleration equations to determine the distance without using a  $v-t$  diagram was also possible.

*Answer:* The total distance travelled in the first 40 s of motion is 1050 m

- (ii) Candidates found this part to be quite difficult. In order to explore the motion of the motorcycle it is first necessary to find the total time of travel of the car. It decelerates from a speed of  $30 \text{ ms}^{-1}$  to zero over the remaining distance of 450 m, giving a total time for the car's travel of 70 seconds. Hence the motorcycle travels for 50 seconds. Since the motorcycle travels at constant speed for 30 seconds then it is accelerating and decelerating in total for 20 seconds and from the given information that  $a = 3d$  then this is split as 5 seconds for acceleration and 15 seconds for deceleration. One possible method for finding the required acceleration is to determine the speed  $V \text{ ms}^{-1}$  of the motorcycle and this can be achieved by using the fact that the motorcycle travels the same distance as the car, namely 1500 m where  $1500 = \frac{1}{2}V(30 + 50)$  and this equation gives  $V = 37.5$ . It is now possible to evaluate the required acceleration as  $V \div 5$ .

*Answer:* The value of  $a = 7.5 \text{ ms}^{-2}$

- (iii) Many candidates had difficulty with this part, often displaying a velocity-time graph rather than the required displacement-time graph. For the first ten seconds of the car's motion, the velocity changes linearly and so the displacement varies as a quadratic. The next 30 seconds of motion are at constant speed and hence the displacement varies linearly. The final 30 seconds involves constant deceleration and hence the displacement again varies as a quadratic.

*Answer:* A graph with time  $t$  represented on the horizontal axis with annotations from  $t = 0$  to  $t = 70$  and displacement  $s$  represented on the vertical axis with annotations from  $s = 0$  to  $s = 1500$

$t = 0$  to  $t = 10$  is a quadratic graph, starting at  $(0,0)$  concave upwards to  $(10,150)$ . This curve smoothly joins the straight line from  $(10,150)$  to  $(40,1050)$  and this line smoothly joins the section from  $t = 40$  to  $t = 70$  which is a quadratic graph, concave downwards from  $(40,1050)$  to  $(70,1500)$ . Only a sketch is required but it must show the shape in the three distinct time zones and also the relevant points on the axes.



# MATHEMATICS

Paper 9709/43  
Paper 43

## Key messages

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates are also reminded to maintain sufficient accuracy in their working to achieve this level of accuracy in their final answers (e.g. **Question 1(ii)**, **Question 2** and **Question 6 (iii)**).

Candidates are reminded of the importance of a clear and complete force diagram in problem solving and a consideration of all forces when forming either an equation of motion (e.g. **Question 6 (iii)**) or a work / energy equation (e.g. **Question 7(ii)**).

Candidates should take note of key words in each question, e.g. 'one second later' in **Question 4**, 'total distance' in **Question 4**, 'steady speed' in **Question 6(ii)** and 'a further 5 m' in **Question 7(ii)**.

## General comments

The standard of work seen was very varied, including as usual many excellent, well presented scripts. **Question 5(i)** and **Question 6(i)** were found to be the easiest questions whilst **Question 4(ii)** and **Question 7(ii)** provided the most challenge.

## Comments on specific questions

### Question 1

The majority of candidates found this to be a fairly straightforward question and often gained full marks.

- (i) Accurate solutions either combined PE gain and work done against resistance or used  $WD = Fs$  and calculated  $(50g + 25) \cdot 3.5$ . Common errors were to subtract rather than to add, leading to 1660 J or to consider only the PE gain (1750 J) or to consider only the work done against resistance (87.5 J). A few candidates combined energy with force rather than work done using  $(50g \cdot 3.5 + 25)$ .

*Answer:* Work done by the crane is 1840 J

- (ii) Most candidates used  $P = WD/t$  and halved their answer from **part (i)** rather than calculating the speed and then applying  $P = Fv$ . Some solutions were incomplete due to the inclusion of only one of the two forces whilst others did not achieve three significant figure accuracy following premature approximation for the work done.

*Answer:* Power of the crane is 919 W

### Question 2

Candidates generally knew what was required and this question was well answered. The majority resolved horizontally and vertically and then attempted to solve the resulting two equations. Occasional errors occurred when forming the equations, e.g. using 20 instead of 20g or omitting the 20 kg mass or assuming that the tension was the same in both ropes. Most errors occurred during the solution of the equations either from incorrect elimination of one tension, or from premature approximation, e.g.  $\cos 50^\circ = 0.64$  leading to inaccuracy in the tensions found. A few solutions were attempted using either Lami's Theorem or the Sine Rule but not always with the correct three angles.

*Answers:* Tension in rope PA is 306 N    Tension in rope PB is 200 N

### Question 3

- (i) Although the system involved two pulleys rather than one, most candidates were able to apply their knowledge to this situation with the most common method being to form and solve two equations of motion in order to find the acceleration. A sign error in one equation or in the solution of the two equations sometimes resulted in  $a = 10$ . Alternatively, those who formed a single equation in  $a$  sometimes believed  $7g - 3g = 7a$  resulting in  $a = 5.71 \text{ ms}^{-2}$ . Some candidates found the acceleration but then 'forgot' to find the speed of  $P$ .

*Answer:* Acceleration of the particles is  $4 \text{ ms}^{-2}$  Speed of  $P$  is  $1.79 \text{ ms}^{-1}$

- (ii) This part of the question was more difficult and depended on an understanding of the changed situation for  $Q$  once  $P$  reached the floor. Some candidates continued with the acceleration found in **part (i)** rather than  $a = -g$  once the tension in the string disappeared. It was usual to see attempts to find the additional distance travelled (0.16 m) although this was not always used to compare two appropriate distances. Alternatively, candidates attempted to calculate the speed at the pulley using  $v^2 = u^2 + 2as$  sometimes with  $a =$  their **part (i)** value despite the motion under gravity and sometimes with  $s = 0.5 \text{ m}$  rather than  $0.1 \text{ m}$ .

### Question 4

- (i) For some candidates this involved two straightforward applications of  $s = ut + \frac{1}{2}at^2$  leading easily to  $31.25 + 18.75$ . Those who overlooked the one second time lapse and used  $t = 1.5 \text{ s}$  for  $A$  as well as  $B$ , found height =  $11.25 + 18.75 = 30 \text{ m}$ . Some interpreted 'same height' as 'equal distance travelled', and using times  $t$  and  $t \pm 1$  attempted to form an equation in  $t$  which was often later abandoned. The given  $50 \text{ m}$  height was not always helpful to candidates who found inappropriate calculations such as  $\frac{1}{2}(1.5 + 1) \cdot 20 = 50$ .

- (ii) Most candidates knew that the constant acceleration formulae were needed but frequently found this question challenging, partly due to the different starting times for  $A$  and  $B$  but also because many did not recognise the difference between 'total distance' and displacement. A very common solution was  $s = 20(\sqrt{10} - 1) + \frac{1}{2}(-10)(\sqrt{10} - 1)^2 = 19.9 \text{ m}$  with no consideration of whether ball  $B$  was on the way up or on the way down again. Those who interpreted 'total distance' correctly and found a maximum height ( $20 \text{ m}$ ) to combine with a downward distance were not always calculating with a correct time for  $B$ 's motion. Thus, for example,  $26.7 \text{ m}$  was often seen following the use of the values  $t_A = t_B = 3.16 \text{ s}$ .

*Answer:* Length of time for which ball  $B$  is in motion when ball  $A$  reaches the ground is  $2.16 \text{ s}$   
Total distance travelled by ball  $B$  up to the instant ball  $A$  reaches the ground is  $20.1 \text{ m}$

### Question 5

This question was well answered with most candidates recognising that integration was required and many gaining full marks.

- (i) Integration to obtain  $s(t)$  was completed accurately by the vast majority who were usually able to evaluate the displacement for time  $t = 20 \text{ s}$ . Those who did not evaluate  $s(20)$  sometimes wrongly used  $a = 0$  rather than  $v = 0$  when attempting to find the time at which particle  $P$  was at rest leading to  $s(10) = 200 \text{ m}$ .

*Answer:* The distance  $OX$  is  $400 \text{ m}$

- (ii) The integration of  $a = k - 12t$  was more problematic with errors arising from uncertainty about how to integrate  $k$ , e.g.  $k^2/2$  rather than  $kt$ . The constants of integration were also problematic if the initial conditions  $v(0) = 0$  and  $s(0) = 0$  were not recognised. Those who applied only  $s(10) = 400$  were unable to evaluate  $k$  since their equation in  $k$  also contained unknown constants of integration.  
A number of candidates mistakenly found  $k = 128$  having chosen to apply the equation  $s = ut + \frac{1}{2}at^2$  with  $t = 10$  and  $a = k - 12t$ , despite the varying acceleration, and surprisingly sometimes following integration in **part (i)**.

*Answer:*  $k = 48$

### Question 6

This question was straightforward for many candidates and fully correct solutions were common particularly in **parts (i) and (ii)**.

- (i) Most candidates appeared confident in applying Newton's Second Law to achieve the given answer.

*Answer:* Total mass of cyclist and bicycle is 80 kg

- (ii) Candidates often set up a correct implicit equation for  $v$ . A few overlooked 'steady speed' and included ' $ma$ ' while others omitted the component of weight. The answer 6.26 was given in the question and candidates were expected to provide sufficient working, e.g.  $v = 300/(20 + 80g\sin 2^\circ)$ .

- (iii) While many candidates formed a correct equation using Newton's Second Law, it was quite common to find missing terms with either the resistance (20 N) or the component of weight down the plane ( $80g\sin 2^\circ$ ) omitted. Some oversimplified the situation to  $P/v = ma$  or  $P = mav$  leading to  $v = 0.666 \text{ ms}^{-2}$ . Others who would have reached  $0.0666 \text{ ms}^{-2}$  gave their answer as  $0.07 \text{ ms}^{-2}$  correct to one significant figure rather than three significant figures as required.

*Answer:* Acceleration of the cyclist is  $0.0666 \text{ ms}^{-2}$

### Question 7

- (i) This question asked for an 'inequality' for the coefficient of friction between the box and the plane. Many candidates applied  $F = \mu R$  to obtain  $\mu = 0.176$  considering only the case of limiting equilibrium. A variety of incorrect inequalities was seen including  $-0.176 < \mu < 0.176$ .

*Answer:*  $\mu \geq 0.176$

- (ii) This part challenged some of the strongest candidates and fully correct solutions were infrequent. Nevertheless, most were able to gain some credit for calculating the potential energy loss and / or the work done against resistance. The two stages of motion were a key source of error with many candidates tackling only the first 5 m of motion, apparently overlooking the 'further 5 m'. The majority attempted an energy method. However, there were also occasional attempts using only Newton's Second Law which could not gain full credit since the question asked for an 'energy method'. A significant number of candidates included both the work done by the component of weight down the plane and the potential energy loss without recognising this as duplication. The neatest solutions, seldom seen, made use of a single work/energy equation for the full 10 m journey.

*Answer:* The speed of the box after it has travelled a further 5 m is  $2.70 \text{ ms}^{-1}$

- (iii) **Part (iii)** was much better answered with many candidates managing the change to  $20^\circ$  successfully although some maintained  $10^\circ$  when considering the frictional force. Others omitted one of the two force components from their equation of motion.

*Answer:* The acceleration of the box is  $1.63 \text{ ms}^{-2}$

# MATHEMATICS

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Paper 9709/51  
Paper 51

## General comments

The work on this paper was generally well presented with a few exceptions.

A number of candidates lost marks due to prematurely approximating within their calculations leading to an incorrect answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

Most candidates are now using  $g = 10$  as instructed on the question paper.

Occasionally candidates use an incorrect formula. A formula booklet is provided so it is a good idea to keep a check on it.

The easier questions proved to be **1**, **3(i)**, **5(i)** and **7(ii)**.

The harder questions proved to be **2(ii)**, **3(iii)**, **4(iii)** and **7(i)**.

## Comments on specific questions

### Question 1

The question was generally well done.

This question required the candidate to use both  $T = \lambda x/l$  and  $T = mv^2/r$ . Having done this they can be equated to give the final result.

Some poor manipulation on occasions resulted in the wrong answer.

Answer:  $l = 0.2$

### Question 2

This was a question where some candidates used the wrong formula for the centre of mass. Also a number of candidates used areas instead of arc lengths.

- (i) Some candidates simply found the centres of mass of the two parts and then subtracted. The correct answer appeared but from incorrect working. Most candidates attempted to take moments about ACB as required.
- (ii) Too many candidates just said  $\tan \theta = 0.191/0.6$ . This part of the question was worth 4 marks and so candidates should have realised that more work was required. Moments about A should have been attempted before the required angle could be calculated.

Answers: (i) 0.191 m (ii)  $15.3^\circ$

### Question 3

- (i) This part of the question was usually well done.
- (ii) A number of candidates integrated but had  $v = \dots$  not  $v^2/2 = \dots$ . Generally this part was well done.
- (iii) Quite a number of candidates found a numerical value and not an algebraic answer. Too often the direction was not stated.

Answers: (i)  $v dv/dx = 1.2x^2 + 8$  (ii)  $5.17 \text{ ms}^{-1}$  (iii)  $0.5 + 0.75x \text{ N towards O}$

### Question 4

- (i) Quite a number of candidates used  $0.9 \times 1/3$  instead of  $0.9 \times 2/3$  as the distance of the centre of mass of the triangle from AD. Most candidates attempted to take moments about AD using a rectangle and a triangle.
- (ii) A few candidates used  $a + 2a/3$  instead of  $a + a/3$  as the distance of the centre of mass of the triangle from AB.
- (iii) Most candidates attempted to take moments about  $D$  but some used incorrect distances.

Answers: (i)  $0.5 \text{ m}$  (ii)  $7a/9 \text{ m}$  (iii)  $a = 0.75$

### Question 5

- (i) This part of the question required the candidate to find the angle made by the string to either the horizontal or the vertical. The next step was to use Newton's Second Law towards the centre of the circle. This was generally well done.
- (ii) This part of the question required the candidate to do two things. They were required to resolve vertically and to use Newton's Second Law towards the centre with acceleration  $r\omega^2$ , where  $r$  is the radius of the circle.

Answers: (i)  $2.45 \text{ ms}^{-1}$  (ii)  $6.67 \text{ N}$  and  $4.71 \text{ rads}^{-1}$

### Question 6

This type of question in the past often proved to be difficult for many candidates. It was pleasing to see many good attempts.

- (i) The candidate was required to set up a four term energy equation containing 2 EE, 1 KE and 1 PE term. Many candidates had all four terms and often went on to find the correct answer. Some errors occurred when finding the extension of the string.
- (ii) The string becomes slack when the arc length is  $1.2 \text{ m}$  and so by using arc length  $= r\theta$ , we get  $1.2 = 0.9\theta$ . Now another four term energy equation is required.

Answers: (i)  $8.29 \text{ ms}^{-1}$  (ii)  $8.13 \text{ ms}^{-1}$

**Question 7**

- (i) Many candidates realised that two equations could be set up by substituting the two points into the given equation. Solving the two equations proved to be too difficult for many candidates.
- (ii) This part of the question was usually well done.
- (iii) Many candidates struggled with this part of the question. It was necessary to find both the horizontal and vertical velocities and then use Pythagoras's theorem and trigonometry to find the speed and angle of direction.

*Answers:* (i) 63.435 (ii) 0.894 s (iii)  $16.3 \text{ ms}^{-1}$  and  $15.9^\circ$

# MATHEMATICS

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Paper 9709/52  
Paper 51

## General comments

The work on this paper was generally well presented with a few exceptions.

A number of candidates lost marks due to prematurely approximating within their calculations leading to an incorrect answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

Most candidates are now using  $g = 10$  as instructed on the question paper.

Occasionally candidates use an incorrect formula. A formula booklet is provided so it is a good idea to keep a check on it.

The easier questions proved to be **1**, **3(i)**, **5(i)** and **7(ii)**.

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## Comments on specific questions

### Question 1

The question was generally well done.

This question required the candidate to use both  $T = \lambda x/l$  and  $T = mv^2/r$ . Having done this they can be equated to give the final result.

Some poor manipulation on occasions resulted in the wrong answer.

Answer:  $l = 0.2$

### Question 2

This was a question where some candidates used the wrong formula for the centre of mass. Also a number of candidates used areas instead of arc lengths.

- (i) Some candidates simply found the centres of mass of the two parts and then subtracted. The correct answer appeared but from incorrect working. Most candidates attempted to take moments about ACB as required.
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Answers: (i) 0.191 m (ii)  $15.3^\circ$



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- (i) This part of the question was usually well done.
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- (iii) Quite a number of candidates found a numerical value and not an algebraic answer. Too often the direction was not stated.

Answers: (i)  $v dv/dx = 1.2x^2 + 8$  (ii)  $5.17 \text{ ms}^{-1}$  (iii)  $0.5 + 0.75x \text{ N towards O}$

### Question 4

- (i) Quite a number of candidates used  $0.9 \times 1/3$  instead of  $0.9 \times 2/3$  as the distance of the centre of mass of the triangle from AD. Most candidates attempted to take moments about AD using a rectangle and a triangle.
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**Question 7**

- (i) Many candidates realised that two equations could be set up by substituting the two points into the given equation. Solving the two equations proved to be too difficult for many candidates.
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- (iii) Many candidates struggled with this part of the question. It was necessary to find both the horizontal and vertical velocities and then use Pythagoras's theorem and trigonometry to find the speed and angle of direction.

Answers: (i) 63.435 (ii) 0.894 s (iii)  $16.3 \text{ ms}^{-1}$  and  $15.9^\circ$

# MATHEMATICS

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Paper 9709/53  
Paper 51

## General comments

The work on this paper was generally well presented with a few exceptions.

A number of candidates lost marks due to prematurely approximating within their calculations leading to an incorrect answer. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

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Occasionally candidates use an incorrect formula. A formula booklet is provided so it is a good idea to keep a check on it.

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## Comments on specific questions

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The question was generally well done.

This question required the candidate to use both  $T = \lambda x/l$  and  $T = mv^2/r$ . Having done this they can be equated to give the final result.

Some poor manipulation on occasions resulted in the wrong answer.

Answer:  $l = 0.2$

### Question 2

This was a question where some candidates used the wrong formula for the centre of mass. Also a number of candidates used areas instead of arc lengths.

- (i) Some candidates simply found the centres of mass of the two parts and then subtracted. The correct answer appeared but from incorrect working. Most candidates attempted to take moments about ACB as required.
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Answers: (i) 0.191 m (ii)  $15.3^\circ$

### Question 3

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- (iii) Quite a number of candidates found a numerical value and not an algebraic answer. Too often the direction was not stated.

Answers: (i)  $v dv/dx = 1.2x^2 + 8$  (ii)  $5.17 \text{ ms}^{-1}$  (iii)  $0.5 + 0.75x \text{ N towards O}$

### Question 4

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- (iii) Most candidates attempted to take moments about  $D$  but some used incorrect distances.

Answers: (i)  $0.5 \text{ m}$  (ii)  $7a/9 \text{ m}$  (iii)  $a = 0.75$

### Question 5

- (i) This part of the question required the candidate to find the angle made by the string to either the horizontal or the vertical. The next step was to use Newton's Second Law towards the centre of the circle. This was generally well done.
- (ii) This part of the question required the candidate to do two things. They were required to resolve vertically and to use Newton's Second Law towards the centre with acceleration  $r\omega^2$ , where  $r$  is the radius of the circle.

Answers: (i)  $2.45 \text{ ms}^{-1}$  (ii)  $6.67 \text{ N}$  and  $4.71 \text{ rads}^{-1}$

### Question 6

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- (i) The candidate was required to set up a four term energy equation containing 2 EE, 1 KE and 1 PE term. Many candidates had all four terms and often went on to find the correct answer. Some errors occurred when finding the extension of the string.
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Answers: (i)  $8.29 \text{ ms}^{-1}$  (ii)  $8.13 \text{ ms}^{-1}$

**Question 7**

- (i) Many candidates realised that two equations could be set up by substituting the two points into the given equation. Solving the two equations proved to be too difficult for many candidates.
- (ii) This part of the question was usually well done.
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*Answers: (i) 63.435 (ii) 0.894 s (iii)  $16.3 \text{ ms}^{-1}$  and  $15.9^\circ$*

# MATHEMATICS

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<p><b>Paper 9709/61</b> <b>Probability and Statistics 1</b></p>
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## **Key Messages**

Candidates should be aware of the need to work to at least 4 significant figures to achieve the required degree of accuracy. Efficient use of a calculator is expected, but candidates should be encouraged to show sufficient workings in all questions to communicate their reasoning.

## **General Comments**

Candidates would be well advised to read the question again after completing their solution to ensure they have included all the relevant details.

Candidates who presented their work in a clear and ordered fashion, often avoided careless mistakes that were seen on some papers.

**Questions 4 and 7** were generally answered well.

## **Comments on Specific Questions**

### **Question 1**

This was a text book style standardisation question. Candidates should have recognised the need to use the z-value from the critical values table provided. Good solutions were clearly presented, with the algebra involved in rearranging the formula shown. Weak solutions often had errors that occurred as their workings were not presented in an ordered style. A few candidates failed to recognise the notation  $N(20,49)$  and did not square root the variance.

*Answer:* 24.7

### **Question 2**

Good solutions used a probability distribution table to clarify their thinking, and used the results to calculate the expected value of the difference between the two scores. It was unfortunate that many candidates who used this approach failed to complete the calculation. Weaker candidates often successfully completed a probability outcome table, but then did not identify the probability for each outcome and so could gain little credit. A significant number of solutions only calculated the probability of a difference of zero.

*Answer:* 1.94

### Question 3

- (i) Most candidates recognised that for one mark a simple process would be required and multiplied the probabilities successfully. Candidates often stated their answer to 3 decimal places which was equivalent to 2 significant figures, and so was not accurate enough to gain credit. A few solutions involved summing the separate probabilities and achieving an answer greater than 1, which is not possible for a probability and should have indicated to the candidate that there was an error in their work.
- (ii) Good solutions identified that the binomial distribution was appropriate, and used the probability of seeing and not seeing lions in the binomial distribution formula for 12 outcomes. Many solutions omitted the possibility that 0 visitors saw the lions when considering 'fewer than 3'. A number of candidates reversed the probabilities, and calculated 'more than 3 visitors saw the lions'. Candidates do need to work to at least 4 significant figures if they are to obtain a final answer correct to 3 significant figures.
- (iii) This was a simple text book calculation of the mean and variance from the data provided. Most candidates who attempted the question had some success. The most common error was to calculate the standard deviation rather than the variance.

Answers      (i) 0.0727      (ii) 0.889      (iii) 42.5, 6.375

### Question 4

This question was attempted by the majority of the candidates. Part (ii) could be answered without having successfully completed (i).

- (i) Good solutions responded to the prompt 'normally distributed' and used the normal standardisation formula accurately. A simple sketch of the normal distribution often seemed to help candidates identify the correct probability required. As weight is a continuous variable, no continuity correction was required.
- (ii) Although this was a simple arithmetical calculation interpreting the information from the context, many candidates did not divide the total mass of rice available by the mean (average) weight of the packet.
- (iii) Good solutions identified that this was the reverse of the process in (i) and worked through successfully. Many candidates used the positive z-value and so achieved the incorrect value for  $\mu$ .
- (iv) Candidates generally realised that the same calculation as (ii) was required, and repeated the process if they had obtained a value in (iii).

Answers      (i) 0.0093      (ii) 961 or 962      (iii) 1.03      (iv) 971 or 970



### Question 5

- (a) Many candidates found this question challenging. The most common approach was to consider the arrangements of the consonants (P,N,L) and then multiply by the number of ways the vowels (A,E,I) could be inserted between the consonants. 'Arranging' is the key word to indicate that permutations are anticipated within the solution. Most solutions identified the need to remove the effect of repeated letters, so divided by 2! and 3!. Many candidates used 5! or  ${}^5P_5$  to calculate the arrangements of the consonants. However, many candidates did not identify that there were 6 possible positions that the vowels could be placed and so used 4! rather than  ${}^6P_4$ .
- (b)(i) It was encouraging to see that many candidates identified the key word 'choosing' to link with combinations and successfully calculated this problem within the given context.
- (ii) The best solutions identified that subtracting the number of ways that teams could be chosen with both players included from their answer in (i) would give the required outcomes. However, most candidates attempted to sum the possible team combinations that did not include both players, but often made errors in identifying the number of batsmen or bowlers they could choose from for the particular scenario. Where candidates stated the restrictions they were applying they were often more successful and achieved all the required groups.

Answers: (a) 3600 (b)(i) 420 (ii) 180

### Question 6

The best solutions used a tree diagram to clarify the information within the question and to ensure that the correct probabilities were identified for 'Operation T' outcomes.

- (i) Where attempted, most candidates failed to reflect that when a blue pen is moved to the new pocket that there would now be 5 pens to choose from. A large number of solutions used a denominator of 4 throughout.
- (ii) Good solutions recognised that there were 2 possible ways of achieving 1 blue pen in the left pocket after Operation T, using their answer from (i) and the P(R,R).
- (iii) Many candidates identified that this was a question on conditional probability and attempted to use the relevant formula. Good solutions justified clearly their denominator as anticipated at this level. Full credit is not normally given where an answer is simply stated.

Answers: (i) 1/10 (ii) 14/20 (iii) 2/5

### Question 7

The best stem-and-leaf diagrams were often seen within the graph booklet as this assisted candidates to achieve the accuracy expected in the shape of the leaves. The best diagrams using the main booklet were where the stem had been drawn in the middle of the page. This question was attempted by almost all candidates.

- (i) A number of good solutions with a single integer stem, leaves with values equally spaced and a single key including both the units and the factor were seen. However, many keys only contained one of these elements. As this is a diagram, it is acceptable to complete in pencil, so that any errors can be erased and accuracy maintained. If values are crossed out, the new value needs to be in the same place. A number of candidates produced 2 single side diagrams, which could gain little credit.
- (ii) There was a good understanding of the median, even though some candidates ordered the component masses for factory *B* again. There was less certainty about the inter-quartile range. Candidates often calculated accurately the Upper Quartile and then did not use the same process to identify the Lower Quartile. A significant number of solutions stated the range.
- (ii) As in all interpretive statistical questions, candidates were required to put their answer within the context of the question. Good solutions made statements such as 'components in factory *B* are lighter' (which compared the medians) and 'the masses are less spread out in factory *A*' (comparing the range). These comparisons could be based upon the back-to-back stem-and-leaf diagram.

Answers      (ii) median 0.048g IQR 0.015g

# MATHEMATICS

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Paper 9709/62  
Probability and Statistics 1

## Key Messages

Candidates should be aware of the need to work to at least 4 significant figures to achieve the required degree of accuracy. Efficient use of a calculator is expected, but candidates should be encouraged to show sufficient workings in all questions to communicate their reasoning.

Candidates would be well advised to read the question again after completing their solution to ensure they have included all the relevant details.

## General Comments

It was pleasing that many candidates seem to have prepared well for the examination, and many good solutions were seen. There did not appear to be any issue with the accessibility of the questions, nor the time available for the paper.

Candidates who presented their work in a clear and ordered fashion often avoided careless mistakes that were seen on other papers. Some candidates made more than one attempt at a question and then failed to indicate which their final solution was. This is often to the disadvantage of the candidate. Candidates should be reminded that corrections should be made clearly, not overwriting previous answers, as it is often not possible to determine their intention.

Answers to **Questions 1, 3 and 4** were generally stronger than other questions.

## Comments on Specific Questions

### **Question 1**

It was pleasing to see that this opening question was well done by almost all the candidates. They recognised that the situation involved a conditional probability and successfully worked out the numerator and denominator. The best solutions included a clear tree diagram which clarified the values involved.

*Answer:* 0.464

## Question 2

- (i) Candidates need to be aware that if the answer is given, full and clear workings need to be included in their answer if they are to show that the statement is true. This includes reaching the final given answer. Many candidates achieved this successfully, with the most common approach using the combinations method. Where candidates calculated the probabilities for different outcomes, common errors were either not identifying all possible outcomes, or assuming that the items are replaced after being chosen and not reducing the denominator. It was noted that a number of candidates used a tree diagram here to clarify the outcomes. Candidates should realise that if they do not obtain the given answer, they have made an error.
- (ii) Almost all candidates drew a probability distribution table, although not all solutions included the answer provided within the question. Candidates who were successful in (i) repeated their method here. A number of arithmetical slips were noted, but most candidates realised that the probabilities needed to add to 1, so were able to gain some credit.

Answers: (ii)  $P(0) = 84/220$ ,  $P(2) = 27/220$ ,  $p(3) = 1/220$

## Question 3

- (i) Almost all candidates recognised that a binomial approximation was appropriate for this question. However, most solutions considered the two options separately, rather than calculating the probability that Kersley was either asleep or studying as 0.8 before using the binomial approximation. It was encouraging that most candidates interpreted 'at least 6 days' correctly.
- (ii) Again, most candidates decided that the normal approximation was appropriate and applied the formula correctly. A number of solutions were seen which incorrectly used the answer obtained in (i) for the probability of being asleep. Most solutions involved a continuity correction attempt, but a significant proportion used the lower boundary, so effectively excluded the required value. Good solutions often had a sketch of the normal distribution which was used to identify the correct 'area' required, and hence the size of the final probability.

Answers: (i) 0.577 (ii) 0.996

## Question 4

- (i) This part was attempted well by most candidates. Good solutions used a simple sketch of the normal distribution to identify the required region and then determined an appropriate method to obtain the value required. Without this sketch, many candidates incorrectly found the difference between their two answers. A number of solutions did not use the tables to find the probability of  $z < 0.5$ , but stated it was 0.5.
- (ii) Many clear, correct solutions were seen. Candidates need to be aware of how to use the table provided accurately, as simple numerical value errors were often seen and many candidates attempted to use a probability rather than a z-value in their work.
- (iii) Many candidates found this part more challenging, although it is a common task. Most candidates were able to solve the power equation that they generated, but there was much confusion about the value to use for the probability. Many candidates did not respond to the prompt in this question about 'less than 0.003' which required the use of complementary probability.

Answers: (i) 0.570 (ii) 4.91 (iii) 46

### Question 5

Although attempt by almost all candidates, many did not appear to have prepared thoroughly on the requirements of histograms, and many did not proceed past their attempt at calculating the frequency distribution. Weaker candidates often did not make an attempt at (ii) and (iii) which considered other aspects of central tendency knowledge.

- (i) The best solutions used a table to calculate firstly the boundaries of the classes, then the class widths and finally the frequency distribution. The scale was stated for the horizontal axis, and good graphs had a scale that enabled all frequency distribution values to be plotted accurately. Although many candidates used the incorrect class widths to calculate their frequency distribution, most attempted to draw the class bar ends correctly. Good solutions often simplified this by translating the axis slightly to 2500, 12 500, etc. which is acceptable. Candidates must ensure that their axes represent values being used, so if the full values were not used on the horizontal axis, then the units must be included in the label.
- (ii) The best solutions used the table they constructed in (i) to clarify the working for this part. Working lines were added for mid-value and frequency mid-value. Common errors were the use of class widths or class boundaries within the calculation.
- (iii) When attempted, candidates were often more successful in identifying the class containing the median than the lower quartile. Many candidates did identify the class containing the upper quartile. Candidates were expected to refer to the classes in the table rather than any alternative notation.

Answers: (ii) 18 600 (iii) median = 8–12 thousand, LQ = 3–7 thousand

### Question 6

- (i) Many candidates recognised that permutations were required for this part (the key word is 'arranged'), and divided by  $2!2!$  to remove the repeated letters E and N. Some forgot to multiply by 2, which was required as the vowels and consonants could be swapped round. It was unfortunate that a number of candidates solved the question using 3 vowels and 5 consonants from the clarification information provided.
- (ii) Many good attempts were seen to this question. Candidates should be reminded that only non-exact numerical answers should be rounded to 3 significant figures. The clearest solutions considered the number of ways the letters excluding Es could be arranged and then how the Es could be inserted so that they were not next to each other. Some candidates forgot that the Es were repeated so needed to be divided by  $2!$ , as in (i). The alternative approach was to subtract the number of ways the Es were together from the total number of ways that the letters could be arranged. Candidates using this method were often less confident on the number of letters involved, and the repeats that needed to be removed.
- (iii) Many candidates did not recognise the key word 'selected' to indicate that the use of combinations was required. The best solutions stated the possible options and then calculated the number of outcomes before summing to obtain their answer. A small number of candidates anticipated that the total number of arrangements would be manageable with a listing method, and successfully identified all 16 ways but failed to state a final answer.

Answers: (i) 8640 (ii) 725 760 (iii) 16

# MATHEMATICS

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<p><b>Paper 9709/63</b> <b>Probability and Statistics 1</b></p>
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## Key Messages

Candidates should be encouraged to show all necessary workings especially when the answer is given. When drawing graphs, candidates should be encouraged to use a sensible scale that enables accurate readings to be achieved.

## General Comments

Answers to questions **6(i)**, **7(i)** and **7(iii)** were generally stronger than answers to other questions. Candidates would be well advised to read the question again after completing their solution to ensure they have answered as required.

## Comments on Specific Questions

### Question 1

This was a challenging first question. The best candidates appreciated that they did not need to consider men and women separately and produced a neat solution in a couple of lines. Those who considered all the different combinations of men and women produced a large number of possible outcomes and it was unusual for them to achieve the correct answer. Common errors were to add the number of ways with William and Mary to the total number of ways rather than subtract or to find the number of ways with William only and Mary only and not to consider the number of ways with neither.

*Answer:* 196

### Question 2

- (i) Whilst, many fully correct solutions were seen using one of the binomial methods, often the binomial coefficients were missing and the candidates only picked up one mark for  $(1/3)^4$  or  $(2/3)^4$ . Those who tried a listing method rarely found all the possible options.
- (ii) This question was answered well. Most candidates correctly stated a way of making the total of 5 (e.g. 1,1,1,2) and many correctly calculated the probability of one of these ways. The most common error was not to realise that there were four possible ways of making a total of 5.

*Answers:* (i) 11/27 (ii) 4/81

### Question 3

- (i) The strongest candidates realised that if they are asked to 'show that' an answer is a particular value they need to fully explain their working. Correctly listing the numbers proved to be the more successful method but candidates were required to conclude and state that there was a total of twelve numbers to gain full marks. If they chose the 'explaining' method, only those who correctly stated that an odd number needed to end with a 5 or a 7 could gain full marks.
- (ii) Those candidates who appreciated that the number could have 1, 2, 3 or 4 digits generally gained full marks. A common error was to assume that the number had to have 4 digits.

Answers: (ii) 32

### Question 4

- (i) Stronger candidates appreciated that they could extract and use the relevant numbers from the table and write down the answer. A common error was to use 90 as the denominator i.e. the total number of cars manufactured in Korea rather than the total number of cars. Others tried to multiply the probabilities rather than add them.
- (ii) This part of the question was done correctly by most candidates – the most common error was to give the probability of being manufactured in Japan rather than not manufactured there.
- (iii) Most candidates knew how to determine whether a pair of events were independent or not and many of them produced correct and clearly expressed solutions. To achieve full marks, a candidate needed to state separately the value of the probability of  $X \cap Y$  and the value of the product of the probabilities of  $X$  and  $Y$  before they compared them. The best candidates stated the probability of the intersection as  $32/250$ , clearly showing they had not multiplied  $8/25$  and  $2/5$ . A few candidates did not realise their two values were equal as they did not simplify their values and came to the wrong conclusion.

Answers: (i)  $64/250$  (ii)  $190/250$  (iii) Independent

### Question 5

- (i) Strong candidates used a simple scale on both axes, labelled their axes with 'cumulative frequency' and 'height in cm' and understood that both graphs were required to be on the same set of axes. They also knew to plot the points above the upper boundaries of the groups and to include (140,0) for the girls' graph and (150, 0) for the boys. Only a few incorrectly interpreted the boundaries as 140.5 etc. Those who used scales not going up in counts of 10 (e.g. 3 or 6) generally lost marks through inaccuracy.
- (ii) The strongest candidates drew a line on their graph up from 165 and across to meet the vertical axis to illustrate their method. Many forgot to subtract from 60 and found the proportion shorter than 165cm rather than taller. Some candidates gave their answer as a fraction or decimal rather than a percentage as required.
- (iii) As in all interpretive statistical questions, candidates were required to set their answer within the context of the question. The best answers commented on how a Box and Whisker plot allows the reader to easily determine whether boys or girls are taller and which gender has a wider variation in height. No credit was given to those who gave a general text book definition of the role of a Box and Whisker plot.

Answers: (ii) 30%



### Question 6

- (a) (i) This was a well answered question. Most candidates correctly standardised and knew to subtract from one. Only a few incorrectly used a continuity correction or did not subtract from one.
- (ii) Most candidates used the tables correctly and successfully produced a z-value. Most also standardised correctly and equated to their z-value. It was, however, common for them to produce a z-value of 1.281 rather than 1.2815 or 1.282 from the critical values from the tables provided and to forfeit the final accuracy mark.
- (iii) The majority of candidates provided sufficient working and reasoning to 'show' this given answer.
- (b) There were many fully correct solutions seen and most candidates knew that they needed to multiply probabilities by prices and sum. Those who put their information into a probability distribution table were most likely to arrive at the correct answer. Some did not go on to multiply by 100, the number of bananas. Candidates who gave an answer in dollars rather than in cents, as in the question, needed to make this clear in their final answer. Common errors were using probabilities that did not sum to one and premature rounding.

Answers: (i) 0.136 (ii) 214g (b) 1930 (\$19.30)

### Question 7

- (i) The majority of candidates recognised the need to use the binomial distribution and produced the correct answer. The most common error was to leave out the  ${}^7C_2$  from the formula and some candidates answered the question for 2 or fewer days.
- (ii) This question proved to be one of the most challenging in the paper. A large number of candidates incorrectly calculated the individual probabilities of rice on two days, pasta on two days and potato on one day and then either added or multiplied these three probabilities together. Others correctly multiplied the probabilities together with powers summing to five but only the strongest candidates recognised the need to then multiply by  $5!/2!2!$ , the number of ways that the described outcome can occur.
- (iii) A pleasing number of candidates produced a fully correct solution to this question. They recognised the need for a normal approximation and a continuity correction. The most common errors were to forget the continuity correction or to subtract their probability from one.

Answers: (i) 0.124 (ii) 0.0253 (iii) 0.933 (or 0.944)

# MATHEMATICS

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Paper 9709/71  
Paper 71

## Key Messages

There were many places on this paper where answers indicated that candidates had not read the question carefully (see below). It is important that candidates are aware that a significant number of marks can be lost by failure to read the scenario and the questions posed properly.

Candidates need to know how to round answers to 3 s.f.

In a 'show that' question, all steps in the calculation need to be shown (see comments below **on Question 6(ii)(a)**)

## General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. In general, candidates scored well on **Questions 3, 4(ii) and 6(ii)(a) and (b)** whilst **Questions 5(b) and 7(ii)** proved particularly demanding.

Most candidates kept to the required level of accuracy; though, particularly on **Question 3**, candidates lost marks for giving final answers to less than three significant figure accuracy; errors could have been made here due to confusion between 3 s.f. and 3 d.p.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

## Comments on specific questions

### **Question 1**

This was a straightforward introductory question testing the calculation of 'unbiased' estimates of the population mean and variance. Many candidates found the correct answers but there were also many who confused the two formulae for the unbiased estimate of the population variance; this has been the case on numerous occasions in the past. There were also candidates who used 16 (the question was about 16-year old males) as their value of  $n$  rather than the sample size of 8. It is important to read the question carefully in order to pick out correct and relevant information.

Answer: 63.5                      14.6

## Question 2

This was not a particularly well attempted question. In part (i) a common error was to state  $H_1$  as  $p > 1/6$  rather than  $p < 1/6$ , and in parts (ii) and (iii) some candidates attempted to use a Normal Distribution. In (ii) it was important that the comparison with 0.05 was shown. To find the smallest  $n$  in (iii), the method that most candidates used was to solve an inequality in  $n$  which led to an answer of 16.4, meaning that the smallest  $n$  was 17; some candidates, having correctly found 16.4, then thought that the smallest  $n$  would be 16 (possibly because of a failure to reverse the inequality sign when dividing by  $\ln(5/6)$ ). There were a few candidates who found the value of  $n$  by trying particular values (16 and 17).

Answer:  $H_0: P(6) = 1/6$     $H_1: P(6) < 1/6$   
0.065 > 0.05  
17

## Question 3

This question was well attempted. Most candidates found the correct value of 1.2 for  $\lambda$  and did the correct calculation to find 'more than 3'. Common errors included the omission of the term  $P(3)$  in their calculation of  $1 - P(0,1,2,3)$  meaning that they calculated '3 or more' rather than the required 'more than 3'. It is important for candidates to interpret the wording carefully on questions such as these. There were also some candidates who wrote their answer to 3 d.p. rather than 3 s.f. (perhaps caused by a lack of understanding of significant figures). In part (ii) many candidates used the correct distribution,  $N(216,216)$ , and common errors here included either lack of, or incorrect use of, a continuity correction.

Answer: 0.0338  
0.0478

## Question 4

Part (i) was not always well attempted. Many candidates seemed to be finding the mean and standard deviation of  $X + Y$ , rather than a mean and standard deviation of the total amount of milk ( $X$ ) in 4 randomly chosen weeks. Another common error was to give a variance as the answer and not a standard deviation as was requested in the question. Both of these errors indicate the importance of reading the question carefully. Part (ii) was well attempted with many candidates finding the required probability.

Answer: 6080   106  
0.990

## Question 5

The confidence interval in part (a) was generally well attempted, though some candidates used incorrect  $z$  values. Part (b) was not so well attempted. Candidates mostly attempted this part of the question by finding  $\sigma$  for the new sample, then using this to find the new  $z$  value and finally the  $z$  value to the value of  $\alpha$ . Some candidates successfully found  $z$  but very few were able to use this to find the correct  $\alpha\%$ . Finding  $n$  in (ii) was done more successfully, but again by using their calculated value of  $\sigma$ .

Answer: 61.5 to 64.5  
67.3  
200

### Question 6

This was, in general, a well attempted question; with part **(ii)(a)**, in particular, being a good source of marks for many candidates. However, there were some candidates who did not show all stages in their working out; as this was a 'show that' question, marks can, and were, withheld for this. There were many correct, and partially correct, answers to part **(i)**, and whilst part **(ii)(b)** was not always fully correct, a credible attempt was made by a good proportion of candidates. Errors included integrating from 0 to 2.4 then omitting to calculate 1 minus their answer. Part **(ii)(c)** was not well answered; there appeared to be a lack of understanding of the situation with most candidates thinking that they needed to do a calculation. The question clearly said 'write down' the probability, indicating that no calculation was necessary if the situation was fully understood. The calculations attempted by many candidates gave incorrect probabilities, often with incorrect answers greater than 1.

Answer:  $m_x, m_y, m_z, m_w$   
0.590  
1

### Question 7

Part **(i)** of this question was reasonably well attempted. However, some candidates either omitted the hypotheses completely or did not define them fully. Calculation of the test value was generally done well, but the comparison (required to justify the conclusion made) was not always done, or not done correctly. As has been mentioned on previous occasions, the comparison needs to be clearly shown as an inequality statement or on a clearly labelled diagram. A statement such as '-1.667 is not in the critical region' is not sufficient, there must be a clear and valid comparison shown of either z values or an area to justify the conclusion made. The conclusion to a significance test should be made in the context of the question with a 'non-definite' statement (as below). Part **(ii)** was not well attempted. There were very few candidates who realised that the first step in finding the probability of a Type II error here was to find the critical value, and then use this to find the required probability. Part **(iii)** was sometimes correctly answered, but there were many candidates who did not answer in the context of the question. A text book definition of a Type II error is not acceptable.

Answer: No evidence that the population mean time has decreased  
0.0567  
Concluding that the mean time has not decreased when in fact it has

# MATHEMATICS

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Paper 9709/72  
Paper 72

## Key Messages

There were many places on this paper where answers indicated that candidates had not read the question carefully (see below). It is important that candidates are aware that a significant number of marks can be lost by failure to read the scenario and the questions posed properly.

Candidates need to know how to round answers to 3 s.f.

In a 'show that' question, all steps in the calculation need to be shown (see comments below **on Question 6(ii)(a)**)

## General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. In general, candidates scored well on **Questions 3, 4(ii) and 6(ii)(a) and (b)** whilst **Questions 5(b) and 7(ii)** proved particularly demanding.

Most candidates kept to the required level of accuracy; though, particularly on **Question 3**, candidates lost marks for giving final answers to less than three significant figure accuracy; errors could have been made here due to confusion between 3 s.f. and 3 d.p.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

## Comments on specific questions

### **Question 1**

This was a straightforward introductory question testing the calculation of 'unbiased' estimates of the population mean and variance. Many candidates found the correct answers but there were also many who confused the two formulae for the unbiased estimate of the population variance; this has been the case on numerous occasions in the past. There were also candidates who used 16 (the question was about 16-year old males) as their value of  $n$  rather than the sample size of 8. It is important to read the question carefully in order to pick out correct and relevant information.

Answer: 63.5                      14.6

## Question 2

This was not a particularly well attempted question. In part (i) a common error was to state  $H_1$  as  $p > 1/6$  rather than  $p < 1/6$ , and in parts (ii) and (iii) some candidates attempted to use a Normal Distribution. In (ii) it was important that the comparison with 0.05 was shown. To find the smallest  $n$  in (iii), the method that most candidates used was to solve an inequality in  $n$  which led to an answer of 16.4, meaning that the smallest  $n$  was 17; some candidates, having correctly found 16.4, then thought that the smallest  $n$  would be 16 (possibly because of a failure to reverse the inequality sign when dividing by  $\ln(5/6)$ ). There were a few candidates who found the value of  $n$  by trying particular values (16 and 17).

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Answer: 0.0338  
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## Question 4

Part (i) was not always well attempted. Many candidates seemed to be finding the mean and standard deviation of  $X + Y$ , rather than a mean and standard deviation of the total amount of milk ( $X$ ) in 4 randomly chosen weeks. Another common error was to give a variance as the answer and not a standard deviation as was requested in the question. Both of these errors indicate the importance of reading the question carefully. Part (ii) was well attempted with many candidates finding the required probability.

Answer: 6080   106  
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## Question 5

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Answer: 61.5 to 64.5  
67.3  
200

### Question 6

This was, in general, a well attempted question; with part **(ii)(a)**, in particular, being a good source of marks for many candidates. However, there were some candidates who did not show all stages in their working out; as this was a 'show that' question, marks can, and were, withheld for this. There were many correct, and partially correct, answers to part **(i)**, and whilst part **(ii)(b)** was not always fully correct, a credible attempt was made by a good proportion of candidates. Errors included integrating from 0 to 2.4 then omitting to calculate 1 minus their answer. Part **(ii)(c)** was not well answered; there appeared to be a lack of understanding of the situation with most candidates thinking that they needed to do a calculation. The question clearly said 'write down' the probability, indicating that no calculation was necessary if the situation was fully understood. The calculations attempted by many candidates gave incorrect probabilities, often with incorrect answers greater than 1.

Answer:  $m_x, m_y, m_z, m_w$   
0.590  
1

### Question 7

Part **(i)** of this question was reasonably well attempted. However, some candidates either omitted the hypotheses completely or did not define them fully. Calculation of the test value was generally done well, but the comparison (required to justify the conclusion made) was not always done, or not done correctly. As has been mentioned on previous occasions, the comparison needs to be clearly shown as an inequality statement or on a clearly labelled diagram. A statement such as '-1.667 is not in the critical region' is not sufficient, there must be a clear and valid comparison shown of either z values or an area to justify the conclusion made. The conclusion to a significance test should be made in the context of the question with a 'non-definite' statement (as below). Part **(ii)** was not well attempted. There were very few candidates who realised that the first step in finding the probability of a Type II error here was to find the critical value, and then use this to find the required probability. Part **(iii)** was sometimes correctly answered, but there were many candidates who did not answer in the context of the question. A text book definition of a Type II error is not acceptable.

Answer: No evidence that the population mean time has decreased  
0.0567  
Concluding that the mean time has not decreased when in fact it has

# MATHEMATICS

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Paper 9709/73  
Paper 73

## Key messages

As well as showing the potential score for a question part, the number of marks allocated to that part is an indication of the steps required in the method of solution. Many candidates followed this guide in **Question 6(i)** and found the unbiased estimates and used these in calculating the confidence interval. Also, some candidates showed all of the necessary steps in **Question 5(ii)**. Other candidates showed only some of these steps in this question part and so did not gain full marks.

## General comments

The combination of independent normal variables and the use of the normal distribution of means of samples were dealt with well by many candidates, as was shown in their solutions to **Question 3**.

The adaptation of Poisson variables and the use of an appropriate normal distribution were handled well by many candidates, as was shown in their solutions to **Question 7(ii)**.

## Comments on specific questions

### Question 1

The majority of candidates answered this question correctly, writing down the Poisson terms and calculating the sum of the three terms. Some candidates added the extra term  $P(X = 3)$ . Other candidates gave only this single term. Other candidates found the complementary probability  $P(X \geq 3)$ , namely  $1 - 0.321$ .

*Answer:* 0.321

### Question 2

Many candidates gave suitable answers for this random number question.

- (i) The required student number was found by multiplying the given random number by 150 and rounding up to the next whole number. So just 58.8 was not enough and nor was 60.

*Answer:* 59

- (ii) The required random number was found by dividing the given student number by 150 or by dividing a number  $n$  from the range  $103 < n \leq 104$  by 150.

Thus any decimal value  $x$  from the range  $0.687 \leq x \leq 0.693$  given to 3 significant figures was acceptable.

*Answer:* most likely answer 0.693



- (iii) Many candidates realised that with this system there was the possibility of repetition of the calculated student number which would result in fewer than five numbers for the sample. Even different, but similar, random numbers could have this effect. Some candidates gave examples of this.

It was incorrect to suggest that some values might be more than 150.

The question asked whether five students would be produced by the system, so it was not relevant for candidates to discuss whether or not five students was a suitable quantity. Comments such as “five people were not representative of all the 150 students” were not acceptable.

*Answer:* possible repeats

### Question 3

Many candidates produced very clear and accurate methods by finding the mean and variance of the race time (178, 41.49) and then using the normal distribution  $N(178, 41.49/15)$  of the means of samples of size 15.

Some candidates added the standard deviations instead of the variances; some marks could still be gained.

Other candidates omitted the essential  $\sqrt{15}$ . Other candidates found the wrong area under the normal curve.

This error usually happened when candidates began their standardisation with  $(178 - 175)$  instead of the correct  $(175 - 178)$ . Also a simple diagram can help with choosing the correct area. A continuity correction was not required.

*Answer:* 0.0356

### Question 4

This question required the sample size to be found from the information given about the significance of the test at different levels. Many candidates handled these calculations well, using both of the significant values ( $z = 1.645$  and  $z = 1.96$ ) to find the range of possible values of  $n$ . The precise set of values for the sample size required the list of the integers 11, 12, 13, 14, 15. Some candidates left their answer as an inequality such as  $10.8 < n < 15.4$  or  $11 \leq n \leq 15$  which were not sufficient, or  $11 < n < 15$  which was incorrect.

Some candidates used only one of the significant values. This gave only a partial answer. Other candidates used an incorrect  $z$ -value such as 2.326.

*Answer:* the set of possible values of  $n$  was 11, 12, 13, 14, 15

### Question 5

- (i) For the hypotheses it was necessary to use  $P(\text{free gift})$  or to use  $p$ .

*Answer:*  $H_0: p = 0.3$   $H_1: p < 0.3$

- (ii) The sums of two specific sets of Binomial terms were required to establish where the Type I error occurred. These were  $P(0, 1, 2) = 0.0355$  and  $P(0, 1, 2, 3) = 0.107$ . Also it was necessary to compare these two probability sums with the significance level 0.05. These comparisons needed to be written down as part of the solution. As  $P(\leq 2)$  was less than 0.05 and  $P(\leq 3)$  was greater than 0.05 the critical region was  $X = 0, 1, 2$  and the probability of the Type I error was 0.0355. Some candidates wrote out this full solution and gained full marks. Some candidates omitted the comparisons with 0.05. Other candidates found only the first sum. These partial answers did not gain full marks. Another quite common error involved finding  $P(\leq 5) = 0.416$  after observing that the mean of the binomial distribution was 6.

*Answer:* 0.0355

- (iii) The test could be carried out by using either probabilities or the critical region. For the probability method it was necessary to compare the “tail”  $P(\leq 3) = 0.107$  with 0.05. It was not sufficient to use just the single term  $P(3)$ , unless a full explanation involving the tail was included.

For the critical region method it was necessary to compare the sample result (3) with the critical region.

For either method a clear conclusion was needed such as the examples listed on the mark scheme. Many candidates gave their conclusion in an appropriate form, using “no evidence that ...” or equivalent.

*Answer:* no evidence to reject the claim that 30% of packets contain a free gift

### Question 6

- (i) Many candidates produced very clear and accurate solutions for the confidence interval. These candidates used the sample results to find the unbiased estimates for the population mean (3.4) and variance (2.02(0202) or 200/99). These values and the variance  $\sigma^2/n$  were then used to find the confidence interval. Some candidates used the sample mean and variance instead. Some marks could still be gained. Other candidates used an incorrect z-value or omitted some of the square roots. A few candidates found the limits 3.12 and 3.68, but did not express their answer as an interval.

*Answer:* 3.12 to 3.68 or ( 3.12 , 3.68 )

- (ii) It was necessary to note that the mean of the values produced by the machine should be 3 as the values were supposed to be equally likely. Then candidates needed to observe that 3 was not contained in the confidence interval and so the machine was probably not working properly. Some candidates stated that 3.4 was in the confidence interval and concluded that the machine was working properly. Other candidates suggested that the machine might not be working correctly because it could only generate values between 3.12 and 3.68. Both of these were incorrect deductions.

*Answer:* the machine was probably not working properly

### Question 7

- (i) It was necessary to find the probabilities for the men and the women separately and then to multiply these two probabilities. Many candidates did this correctly. Some candidates found  $1 - P(\leq 2)$  for the men instead of  $1 - P(\leq 1)$  and found  $1 - P(\leq 3)$  for the women instead of  $1 - P(\leq 2)$ . Other candidates added the two probabilities instead of multiplying them. A few candidates left the two probabilities as separate answers. Other candidates attempted to combine the two Poisson parameters as  $\lambda = 2.5$  and then found  $1 - P(\leq 4)$ . This process was incorrect.

*Answer:* 0.0505

- (ii) Many candidates answered this part efficiently and accurately. For this 1-hour period the parameters for the men and the women were combined to give  $\lambda = 12 + 18 = 30$  for the total number of people. This value was large enough to enable the use of the approximating normal distribution  $N(30, 30)$ . The continuity correction to 35.5 was needed. Some candidates just used 35. Some candidates used 36.5 as the incorrect continuity correction. Some marks were still gained for these attempts.

*Answer:* 0.842

**Question 8**

- (i) Sensibly candidates inspected the diagrams and listed their order of the standard deviations as the question asked and did not attempt some form of calculation.

*Answer:*  $\sigma_x, \sigma_z, \sigma_y, \sigma_w$  or X, Z, Y, W

- (ii) (a) As the answer was given, it was necessary to show all of the steps fully. In particular, having integrated to show that the variance was 5.4, candidates needed to state that the standard deviation was  $\sqrt{5.4}$  or equivalent for their last step before the answer 2.32. Some candidates did not do this.

The variance calculation involved subtracting the square of the mean. Candidates needed to state that the mean was zero (by symmetry) – the quicker way – or to integrate to find this – the longer way. Some candidates omitted this and so did not score full marks.

The answer was given.

- (b) Many candidates answered this part correctly by integrating  $f(x)$  between the limits 2.32 and 3. The precise value was obtained by using the limit as  $\sqrt{5.4}$  to sufficient figures rather than 2.32. Either 0.268 or 0.269 was accepted for the answer mark.

Some candidates tried to use a normal distribution. A few candidates tried to find the median. These attempts were not acceptable.

*Answer:* 0.268

- (c) As  $2\sigma_x$  was greater than the upper limit 3 for  $f(x)$ , the probability was zero.

Some candidates tried to use a normal distribution here as well.

*Answer:* 0